Chapter 5

Balancing Risks and Returns:
Three Theoretical Insights

This chapter looks at the three basic ideas about balancing the investment returns that we seek against the risks that are likely to be incurred — portfolio diversification, correlation of asset risks and returns, and random fluctuations in stock returns — that have become the foundations of modern finance theory.

The chapter is divided into three sections, therefore. The first section examines the Markowitz efficiency frontier and portfolio, the merits of global diversification, and the implications of portfolio diversification for the individual investor. This central idea in modern finance theory basically seeks to construct a portfolio of financial assets such that the returns to the investor are maximized for given risk levels. The second section then looks at the Capital Assets Pricing Model, which relates the riskiness of an asset to its returns, the limitations of this model, and the alternatives to it, such as the Arbitrage Pricing Theory. The third section examines what has now emerged as the most controversial aspect of modern financial theory: whether the markets are indeed informationally efficient or inefficient, and the implications of this debate to the investor.

As we show here, indexing and dollar cost averaging make sense to a passive investor, irrespective of whether the market is informationally efficient or not.
5.1 Portfolio Diversification

5.1.1 Why Diversify?

Remember your mother’s admonishment: “Don’t put all your eggs in one basket!” This is what diversification is all about. The historical evidence examined in the previous chapter clearly points to the superior performance of the equities market, on the whole and in the long run, over the other investment instruments like bonds, commodities and real estate, in nominal as well as in real terms. But the fact that we focused on the whole market, and showed that these returns follow the normal distribution model reasonably well, also suggests that there are equities whose returns have surpassed our whole market mean statistics for long periods. The ‘high-fliers’ equities have produced returns superior to the market average, for instance (section 2.2). Their volatility can be often unnerving however. Indeed, a basic axiom of the market is that the greater the return, the greater will the volatility be. Exhibit 5.1 illustrates this graphically by comparing the annualized returns and volatility for Dow Jones Industrial Index and its components for the 1980-2000 period. Notice the direct overall relation between risk or volatility and return here, i.e., the larger the return the greater the volatility.

Two factors are particularly noticeable here: (a) the better the return the greater the volatility, and (b) the volatility of a portfolio (the Dow) is far less than the volatility of its components. As for (a), while a large volatility itself does not guarantee a large return, this direct relation between risk and return is well supported by the market’s overall history. Based on the CRSP\(^2\) database at University of Chicago, for instance, the average annual return on

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Exhibit 5.1

Higher returns involve higher risks, as this graph of risk versus returns for the Dow and its components shows. These returns are in nominal terms, i.e., unadjusted for inflation, and include dividends. They cover the Jan 1995 - Dec 1999 period, and have been annualized from the monthly data.
small-company stocks during 1926-98 was 17.4%, with a standard deviation of 33.8%. The annual returns for large-company stocks averaged 13.2%, with a standard deviation of 20.3%, during the same period. The market volatility has not stayed the same through its history, of course, as was discussed in the preceding Chapter. A direct relation between returns and volatility has persisted throughout history, nonetheless. But this does not mean that an investor seeking to maximize the returns needs to helplessly resign to the prospects of a bumpy ride through the market’s swings. Rather, a savvy investor would exploit the fact that the volatility of an index or a portfolio is less than the average of the volatility of its individual components. This is what we see in Exhibit 3.1, that the return on the Dow has been far less volatile than the returns on its components.

5.1.2 Markowitz and Portfolio Diversification

Harry Markowitz was the first to show exactly how a portfolio of suitably chosen stocks can reduce volatility to the level that an investor can be comfortable with but without having to sacrifice the returns significantly. His basic idea was simple and straightforward. Aside from the random fluctuations, prices of individual securities also reflect the risks that are specific to the particular company, industry and the market. While market risk would affect all securities, albeit to varying degrees, industry or sector risk would affect only the companies in that sector and company risk would be specific to the individual company. Combining two or more assets with uncorrelated variability thus means diversifying the company risk away, and diversifying the industry or sector risk as well if these equities represent different industries or sectors.

To understand this, let us consider a portfolio of two stocks, X and Y, and suppose that the proportion of total wealth invested in stock X is \( w_x \) and that in stock Y is \( w_y \), i.e., \( w_x + w_y = 1 \) or \( w_y = (1 - w_x) \). Let the historic average returns be \( r_x \) and \( r_y \) for these two stocks and the corresponding standard deviation (the designated measure of volatility or risk) values \( \sigma_x \) and \( \sigma_y \). Let us also accept the continued validity of normal distribution and expect this historic pattern to continue into the future. If we denote the expected returns by \( E(r_x) \) and \( E(r_y) \) for the two stocks, respectively, then the expected return on this portfolio, \( E(r_p) \), is

\[
E(r_p) = w_x E(r_x) + w_y E(r_y)
\]

\[
= w_x E(r_x) + (1 - w_x) E(r_y)
\]

(5.1a)

i.e., the return on a portfolio is the weighted sum of returns on its component securities.
Computing the portfolio’s variance is not as straightforward a matter, however. With \( \sigma_x^2 \) and \( \sigma_y^2 \) as the individual variances of the returns of our two stocks, the portfolio variance, \( \sigma_p^2 \), is

\[
\sigma_p^2 = w_x^2 \sigma_x^2 + w_y^2 \sigma_y^2 + 2 w_x w_y \sigma_{xy} \\
= w_x^2 \sigma_x^2 + w_y^2 \sigma_y^2 + 2 w_x w_y \rho_{xy} \sigma_x \sigma_y
\]  

(5.1b)

Here, \( \sigma_{xy} = \rho_{xy} \sigma_x \sigma_y \) is the covariance of X and Y, and \( \rho_{xy} \) their coefficient of correlation. This coefficient tends to –1 if the two stocks move in opposite directions, +1 if they move exactly in tandem and 0 if they are uncorrelated altogether, i.e., \(-1 \leq \rho_{xy} \leq +1\). It then follows from equation (5.1b) that \( \sigma_p \leq (w_x \sigma_x + w_y \sigma_y) \), i.e., the volatility of a portfolio comprising any two stocks is always smaller than the weighted sum of the stocks’ individual volatilities.

Obviously, we can even eliminate the portfolio volatility altogether, by combining two stocks X and Y such that \( \rho_{xy} = -1 \), and setting \( w_x = \sigma_y / (\sigma_x + \sigma_y) \). That dream may not be worth chasing, however, as two stocks with \( \rho_{xy} = -1 \) are harder to find than those with \( \rho_{xy} = +1 \), particularly if both must have a history of giving better returns than the market. Besides, the variability of returns, whether for the whole market or for individual stocks, hardly ever stays steady for long.

Practical examples of how volatility is reduced when an investment portfolio comprises two or more stocks are not hard to find. Take Microsoft (MSFT) and Home Depot (HD), two of the Dow high-fliers, for instance. Based on monthly returns over a 15-year period, from April 1, 1986 to March 31, 2001, the geometric annual returns have averaged 37.71% for Microsoft, with a standard deviation of 40.92%, and the corresponding numbers for Home Depot are 32.07% and 32.01%, respectively. While these returns are certainly impressive, their volatility too is rather high (Exhibit 5.2), and must have given their investors many anxious moments and sleepless nights. The weak but positive correlation (Exhibit 3.3) of their returns, with the correlation coefficient of 0.39, is apparently fortuitous. The period covered here includes the entire bull-run of the market, from the crash of 1987 to the bear market of 2000-01 and these two stocks have been amongst the top performers of this period.

Setting Microsoft as stock X and Home Depot as stock Y, we thus have \( r_x = 37.71\% \), \( \sigma_x = 40.92\% \), \( \sigma_y = 32.07\% \), \( \sigma_y = 32.01\% \) and \( \rho_{xy} = 0.3896 \). These numbers can be now plugged into the equations (3.1a) and (3.1b), so as to estimate the expected returns and corresponding variances for portfolios with varying proportions \( w_x \) and \( w_y \) of the two stocks. Suppose, for instance, that we wish to estimate \( E(r_p) \) and \( \sigma_p^2 \) for a portfolio with 30% Microsoft and
Exhibit 5.2:
The monthly price changes (left) and corresponding statistical distribution (right) for Microsoft (MSFT) and Home Depot (HD) common stocks. The portfolio shown here is the minimum-variance portfolio computed from equation (5.1b).

Exhibit 5.3
The monthly geometric returns for Microsoft (MSFT) and Home Depot (HD) stocks are weakly, but positively, correlated. Their coefficient of correlation is 0.3896. The April 1986 – March 2001 period has generally been one of rising prices, however, and may well account for this correlation.

70% Home Depot stocks. Then, \( w_x = 0.3 \) and \( w_y = 0.7 \), so that equations (5.1a) and (5.1b) yield
\[
E(r_p) = 0.3 \times 0.3771 + 0.7 \times 0.3207
\]
\[
= 0.3376 = 33.76% \quad (5.2a)
\]
and
\[
\sigma_p^2 = (0.3)^2 \times (0.4092)^2 + (0.7)^2 \times (0.3201)^2
\]
\[
+ 2 \times 0.3 \times 0.7 \times 0.3896 \times 0.4092 \times 0.3201
\]
\[
= 0.01507 + 0.05021 + 0.02143 \quad (5.2b)
\]
\[
= 0.08671
\]
so that
\[
\sigma_p = \sqrt{0.08671} = 0.2945 = 29.45%
\]

To understand what benefits this accomplishes, recall that a typical characteristic of the normal distribution curve of Exhibit 2.32 is that a little over two-thirds (68.26%) of the data would be within one standard deviation from the mean. Using this model, then, the annual returns on a 100% Microsoft portfolio would have 68.26% chance of ranging from a low of –3.21% to a whopping 78.63% whereas a 100% Home Depot portfolio would give between 0.06% to 64.08% annual returns at this probability level. The corresponding numbers for our portfolio of 30% Microsoft and 70% Home Depot stocks are 4.31% and 63.21%. Clearly, we have sacrificed some gains and have reduced the risk substantially.

What we have just calculated is not for the minimum-variance portfolio. The results of these computations for different values of \(w_x\) and \(w_y\) are graphed in Exhibit 5.4. It shows that a portfolio comprising 25% in Microsoft
stocks and 75% in Home Depot stocks would have indeed had the minimum variance here. Note that, in terms of returns, every combination of the two stocks here is superior to investing 100% in Home Depot and all combinations with ≤50% in Microsoft carry either the same or less risk than 100% Home Depot. Indeed, if you were to settle for the same risk as the Home Depot, then investing 50% in Home Depot and 50% in Microsoft would have given you superior returns than carrying only the Home Depot stocks. Also, any combination of the two stocks with more than 50% but less than 100% in Home Depot would have given you better returns at lower risk than if you had owned only the Home Depot stocks in the portfolio.

For comparison, Exhibit 5.5 shows the growth, in nominal dollars by March 31, 2001, of a $1,000 investment of April 1, 1986, for Microsoft, Home Depot, our minimum-variance portfolio and the Dow. With a perfect hindsight, one would have picked Microsoft, of course. But, considering that even the best investor is still human, this example clearly shows that diversification is the way to go.

Exhibit 5.5
In nominal dollars, a $1,000 investment made at the opening of trade on April 1, 1986 would have grown, by March 31, 2001, to $286,000 in the case of Microsoft, $123,000 in the case of Home Depot, and $172,000 for the minimum-variance portfolio computed here.

This idea, that a judicious combination of uncorrelated or poorly correlated stocks greatly lowers the variance of a portfolio while enhancing its return, is what portfolio management is mostly about. Suppose we have any number N of stocks that we wish to combine in a portfolio. We then need to extend equations (5.1a) and (5.1b) from two to the N stock case, to find
their relative proportions, \( w_1, w_2, \ldots, w_N \), and the expected portfolio return \( E(r_p) \) and variance \( \sigma_p^2 \), e.g.,

\[
E(r_p) = \sum_{i=1}^{N} w_i E(r_i) \tag{5.3a}
\]

and

\[
\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} w_i w_j \rho_{ij} \sigma_i \sigma_j \tag{5.3b}
\]

Intimidating though this computational horror may seem, the general scheme here is simple and straightforward, and is illustrated in Exhibit 5.6. Basically, the diagonal boxes in this \( N \times N \) matrix contain the variance terms \( w_i^2 \sigma_i^2 \), and the off-diagonal boxes the covariance terms \( w_i w_j \sigma_{ij} \) (\( = w_i w_j \rho_{ij} \sigma_i \sigma_j \)). Suppose, for simplicity, that we invest equally in all the \( N \) stocks in this portfolio. Then \( w_1 = w_2 = \ldots = w_N = 1/N \). The \( N \) diagonal boxes then add up to \( N \times (1/N)^2 \times \text{average variance} \) and the \( (N^2 - N) \) off-diagonal boxes will add up to \( (N^2 - N) \times (1/N)^2 \times \text{average covariance} \). This gives

\[
\text{Portfolio variance} = \frac{1}{N} \times \text{average variance} + (1 - \frac{1}{N}) \times \text{average covariance} \tag{5.4a}
\]

As \( N \to \infty \), \( (1/N) \to 0 \), and the variance term here tends to vanish. Thus, for large values of \( N \),

\[
\text{Portfolio variance} \Rightarrow \text{average covariance} \tag{5.4b}
\]

**Exhibit 5.6:**

The \( N \times N \) matrix to find the variance of an \( N \)-stock portfolio. The diagonal boxes contain the variance terms \( (w_i^2 \sigma_i^2) \) while off-diagonal boxes contain the covariance terms \( (w_i w_j \sigma_{ij}) \).
The result summarized in Equation (5.4b) has an important implication. It shows that the variance of a portfolio asymptotically approaches the average covariance of returns on stocks comprising the portfolio, no matter how many additional stocks we add to it. There is a practical limit, however, to the number of stocks that one can add to a portfolio. Take three indexes for instance, the Wilshire Total Market Index comprises over 97% of the entire U.S. equities market, in terms of market capitalization, whereas the S&P-500 Index comprises the stocks of 500 largest U.S. businesses, and the Dow comprises only 30. As the volatilities of these three indexes are largely comparable, it is clear that the average covariance of the market never vanishes. We can decompose the risk associated with security price changes into two principal components (Exhibit 5.7). Diversification can only eliminate what is commonly called the ‘unique risk’. This is also known as diversifiable risk because it can be diversified away by adding uncorrelated stocks to a portfolio and is specific either to a single company or to a group of them. The other component of risk, known as the ‘market risk’ or systematic risk, is immune to diversification and comes from the market or economy-wide forces.

**Exhibit 5.7**

Investors in the stock market face two kinds of risk: unique risk tends to be company specific, and can be diversified away, whereas market risk arises from economy-wide forces and is immune to portfolio diversification.

Is there a size limit, then, for the number of stocks in a well-diversified portfolio? The fact that Dow’s 30 stocks mimic the S&P 500 so well, the latter itself being a good proxy for the Wilshire 5000 and the whole market, suggests that 30 may well be a good enough number. This is illustrated in Exhibit 5.8 where we compare the total monthly returns on the Dow and the S&P-500 indexes. The period covered here extends from January 1897 through December 2000. The two sets of returns show a direct relationship, with a correlation coefficient of 0.78. This compares reasonably well with the correlation coefficient of 0.75 for the returns on the Dow versus the Wilshire 5000.
In one of the earliest studies to systematically examine this issue, Fama\(^7\) showed that the volatility of a portfolio drops substantially on the introduction of the first few stocks and practically flattens after about 25 stocks. This result, reproduced in Exhibit 5.9, suggests that unique risk is effectively diversified away when we use a carefully crafted portfolio of 15-25 stocks. Statman\(^8\) has argued that effective diversification does not really require representing the entire market in a portfolio, as a portfolio of no more than about 30 stocks can suffice.

**Exhibit 5.8:**
Total monthly returns on the Dow compared to those on the S&P-500. Despite the rather wide scatter, the relation between the two sets of data here is markedly direct, with a correlation coefficient of 0.78.

**Exhibit 5.9:**
Fama argued that the standard deviation or volatility of a portfolio changes little after about 25 stocks, and that most of the drop in volatility occurs before 10-15 stocks. This suggests that unique risk can be effectively diversified away by carefully selecting a portfolio of 15-25 stocks. International diversification offers an even more efficient way to eliminate unique risk than domestic diversification.
5.1.3 Seeking Diversification Globally

If selecting a few U.S. domestic stocks can help diversify a portfolio, so reducing its risk without sacrificing the returns, then international diversification should achieve similar results with even greater efficiency. This was the rationale for Solnik’s study, the results of which are included in Exhibit 5.10 as “international stocks”. This study found that a portfolio of 5-6 international stocks has the same volatility that a comparable combination of 15 or some domestic (U.S.) stocks would have. This is mainly because systematic risk for a portfolio of domestic stocks is more than double that of the international portfolio. Recent research suggests that a market portfolio of all the world’s endowments would also contribute to societal welfare.

Exhibit 5.10:
The average coefficients of variation of regression of factors show that domestic factors explain almost one-half (46%) of the variations in stock returns worldwide, and more than one-half (55%) of those in the United States.

<table>
<thead>
<tr>
<th>Country</th>
<th>World</th>
<th>Industrial</th>
<th>Currency</th>
<th>Domestic</th>
<th>Joint test of all four factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>0.18</td>
<td>0.17</td>
<td>0.00</td>
<td>0.38</td>
<td>0.39</td>
</tr>
<tr>
<td>Germany</td>
<td>0.08</td>
<td>0.10</td>
<td>0.00</td>
<td>0.41</td>
<td>0.42</td>
</tr>
<tr>
<td>Australia</td>
<td>0.24</td>
<td>0.26</td>
<td>0.01</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.07</td>
<td>0.08</td>
<td>0.00</td>
<td>0.42</td>
<td>0.43</td>
</tr>
<tr>
<td>Canada</td>
<td>0.27</td>
<td>0.24</td>
<td>0.07</td>
<td>0.45</td>
<td>0.48</td>
</tr>
<tr>
<td>Spain</td>
<td>0.22</td>
<td>0.03</td>
<td>0.00</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>United States</td>
<td>0.26</td>
<td>0.47</td>
<td>0.01</td>
<td>0.35</td>
<td>0.55</td>
</tr>
<tr>
<td>France</td>
<td>0.13</td>
<td>0.08</td>
<td>0.01</td>
<td>0.45</td>
<td>0.60</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.20</td>
<td>0.17</td>
<td>0.01</td>
<td>0.53</td>
<td>0.55</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>0.06</td>
<td>0.25</td>
<td>0.17</td>
<td>0.79</td>
<td>0.81</td>
</tr>
<tr>
<td>Italy</td>
<td>0.05</td>
<td>0.03</td>
<td>0.00</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Japan</td>
<td>0.09</td>
<td>0.16</td>
<td>0.01</td>
<td>0.26</td>
<td>0.33</td>
</tr>
<tr>
<td>Norway</td>
<td>0.17</td>
<td>0.28</td>
<td>0.00</td>
<td>0.84</td>
<td>0.85</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.12</td>
<td>0.07</td>
<td>0.01</td>
<td>0.34</td>
<td>0.31</td>
</tr>
<tr>
<td>Singapore</td>
<td>0.16</td>
<td>0.15</td>
<td>0.02</td>
<td>0.32</td>
<td>0.33</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.19</td>
<td>0.06</td>
<td>0.01</td>
<td>0.42</td>
<td>0.43</td>
</tr>
<tr>
<td>All countries</td>
<td>0.18</td>
<td>0.23</td>
<td>0.01</td>
<td>0.42</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Now, diversifying domestically means seeking a representative selection across all the principal market sectors. International diversification then implies diversification not only across different countries and their markets but also across the principal sectors of those markets. It is not surprising, therefore, that unique risk is far more effectively diversified away when we construct an international portfolio than when the portfolio is purely domestic. Domestic factors do not disappear in an internationally diversified portfolio, however. Instead, as is clearly brought out in Exhibit 5.10 in which we reproduce Solnik’s summarization of the results of a factor regression.
study on the relative importance of world, industrial, currency and domestic factors in explaining the stock returns, domestic factors do matter greatly.

An interesting feature of the data in Exhibit 5.10 is that they show currency factor to have the least impact of the four factors considered here. Otherwise, basically, exchange rates vary with interest and inflation rates, by way of International Fisher Effect (Exhibit 5.11), balance of trade and foreign direct investments.

**Exhibit 5.11:** How the domestic interest (denoted by $r_y$ and $r_s$, for Japan and the U.S., respectively) and inflation ($i_y$ and $i_s$) rates affect the spot ($S_{yS}$) and forward ($f_{yS}$) foreign exchange rates.


Two empirical evidences will suffice to explain this. As shown in Exhibit 5.12(a), countries with higher interest rates in a given year tend to have higher inflation rates in the following year and, as shown in Exhibit 5.12(b), the countries with higher inflation rates tend to see their currencies depreciate compared to appreciation of the currencies of countries with lower inflation rates.

**Exhibit 5.12:**

(a) Countries with higher interest rates one year tend to have higher inflation rates next year (top left) and

(b) the countries with high inflation rates tend to see their currencies depreciate (top right).

With rapid globalization, deregulation and privatizations since the 1990s, the country barriers are gradually falling, however. Indeed, today’s world is an increasingly tripartite one (Exhibit 5.13) comprising North America, Europe and Asia. In constructing a globally diversified portfolio, the industry factors are therefore becoming increasingly important, instead of the country factors. This calls for a sector rather than a country approach to portfolio diversification. This has several advantages. As for the investment universe itself, for instance, companies are increasingly hard to classify within a country and can be better compared with their peers in a cross-country approach. Same selection criteria for investment analysis applies within a sector, even when we compare the companies in these sectors across the countries, whereas, were we to focus on a country-by-country analysis then we would need to use a sector-dependent multiplicity of selection criteria for each country. Focus on the sectors also helps us emphasize growth industries, in terms of the investment style that, in the country approach, depends on the country’s economic structure. The sector approach also delegates allocation of capital to company’s management, thus making asset allocation more efficient. This does not, of course, mean that the sector approach has no pitfalls. Nations are sovereign entities, after all, and all politics is essentially local. Sector diversification, for example, does not eliminate currency risk, considered the scourge and the opportunity in international investment. Different accounting principles and practices exacerbate the problems in cross-country comparison of businesses, even when they are in the same sector, as do customs and cultural factors. Information asymmetry remains a problem and investors still prefer to invest

Exhibit 5.13

‘Global’ increasingly implies a tripartite world in which Japan is no longer the sole Asian representative. Indeed, there is not a single country or region that dominates global merchandise production, trade and commerce. This may well signify the past, however, particularly as the impacts of China and India on the world’s merchandise production and hi tech industry, respectively, are hard to gauge as yet.
in their own country and currency. Exhibit 5.14, taken from MSCI-World Index, presents one example of the weights for individual country and sectors in implementing such an approach.

Exhibit 5.14
An example of the sector and country factors in the sector approach for creating an internationally diversified portfolio.

<table>
<thead>
<tr>
<th>Industry</th>
<th>World</th>
<th>USA</th>
<th>Europe</th>
<th>Japan</th>
<th>Pacific</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>6.1%</td>
<td>3.0%</td>
<td>3.0%</td>
<td>0.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Materials</td>
<td>3.7%</td>
<td>1.4%</td>
<td>1.4%</td>
<td>0.6%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Industrials</td>
<td>10.3%</td>
<td>5.8%</td>
<td>2.4%</td>
<td>1.8%</td>
<td>0.3%</td>
</tr>
<tr>
<td>Consumer: (a) Discretionary</td>
<td>14.4%</td>
<td>7.6%</td>
<td>3.4%</td>
<td>3.1%</td>
<td>0.3%</td>
</tr>
<tr>
<td>(b) Staples</td>
<td>7.2%</td>
<td>3.9%</td>
<td>2.7%</td>
<td>0.5%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Health Care</td>
<td>12.1%</td>
<td>7.6%</td>
<td>3.7%</td>
<td>0.7%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Financials</td>
<td>21.2%</td>
<td>9.2%</td>
<td>8.7%</td>
<td>1.9%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Information Technology</td>
<td>13.1%</td>
<td>9.3%</td>
<td>2.2%</td>
<td>1.6%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>8.0%</td>
<td>3.6%</td>
<td>3.7%</td>
<td>0.5%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Utilities</td>
<td>4.1%</td>
<td>2.2%</td>
<td>1.4%</td>
<td>0.4%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Total</td>
<td>100.0%</td>
<td>53.6%</td>
<td>32.5%</td>
<td>11.1%</td>
<td>2.8%</td>
</tr>
</tbody>
</table>

Courtesy: Fabrice Vallat

Note that, however diversified a portfolio is at any given point in time, it needs to be continually monitored and modified. For instance, the data for the 1962-97 period suggest\textsuperscript{15} that the volatility of individual stocks has increased in the recent years while the correlation among stock returns has fallen steadily. As for portfolio diversification, this has two implications: (a) the number of stocks needed to eliminate unique risk has been rising, and (b) portfolio volatility has increased even after this risk has been eliminated. If you thought that international diversification could come to your rescue, think again. Another recent study\textsuperscript{16} has found higher correlation between international equities in bear markets, than in the bull markets. Needless to stress, such a co-movement of equities during the bear markets when the risk of loss is only likely to rise, robs portfolio diversification of its principal attraction as a risk-reduction strategy.

5.1.4 Diversification and the Individual Investor

Our discussions so far have focused on minimizing the variance. But the risk-return profile of a portfolio of stocks is determined as much by the market as by the investor. The market determines the expected return and volatility, \( E(r_p) \) and \( \sigma_p^2 \), for instance, and the investor selects the acceptable combination of risk and return by distributing the wealth between the
selected securities. Stocks are not necessarily the only game in town here, after all. More often than not, three alternatives are typically available — stocks, bonds and money market instruments, for instance. Nor is the choice between domestic and international markets limited to stocks. Exhibit 5.15 shows the risk/return trade-off for an internationally diversified portfolio of bonds. This example, taken from Solnik\textsuperscript{17}, uses 1971-94 data and shows that a U.S. bond investor would have been better off including foreign bonds in the portfolio. Indeed, reducing the U.S. bond content from 100\% (point A) to 70\% (point B) would have reduced volatility the most. What is more, this bond portfolio would have been no more volatile than the 100\% U.S. bond portfolio, but with vastly improved returns, had it comprised 50\% U.S. and 50\% foreign bonds (point C).

\textbf{Exhibit 5.15:}

An internationally diversified portfolio of bonds, based on the 1971-94 data, shows that a U.S. investor would have been better-off adding some foreign bonds to the portfolio.

The points A-E in Exhibit 5.15 are located on a curve that defines all conceivable combinations of the U.S. and foreign bonds here. This curve, or the similar curve in Exhibit 5.4, defines the efficiency frontier — the locus of points where the investor receives the highest rate of return for a given level of risk and assumes the lowest possible risk for a given level of return. Compare any of the points A-E in this Exhibit with F. For the same level of risk as F, point D gives a better return here whereas, for the same level of return as F, point C assumes a far smaller risk.

Minimizing the variance (point B in Exhibit 5.15 or the point in Exhibit 5.4 for 75\% HD and 25\% MSFT) is not the only choice available to an investor, however. Any point on the efficiency frontier could be appropriate. The exact point on the efficiency frontier where the investor decides to be is determined by that individual’s utility curve — this curve represents his or her level of satisfaction with the risk and the return on the investment.

The logic behind the utility or indifference curves $E(U_1) - E(U_3)$ in Exhibit 5.15, with utility (or wealth W) increasing from $E(U_1)$ to $E(U_3)$, is simple. The investor can choose any combination of $E(r_p)$ and $\sigma_p$ on the efficiency frontier for the indifference curve tangential to it, e.g., $E(U_2)$ in
this Exhibit. Hence their designation as indifference curves. The shape of these curves suggests that there is a certain level of substitution between $E(r_p)$ and $\sigma_p$. The utility function that is often used in these studies, and the one used here, is a quadratic function\(^\text{18}\) of the type

$$E(U) = a + br_p + c(r_p^2 + \sigma_p^2) \quad (5.5)$$

where $a$, $b$ and $c$ are constant such that $b > 0$, $c < 0$ and $(b + 2c) > 0$ for all the relevant values of $W$.

The indifference curves implied by this utility function have positive slopes, i.e., $(\partial \sigma_p^2/\partial r_p) = -(b + 2cr_p)/c > 0$ and $(\partial^2 \sigma_p^2/\partial r_p^2) < 0$ at any point along the curve. The marginal rate of substitution (MRS), the rate\(^\text{19}\) at which an individual is willing to trade one good for another while remaining equally well off (this is what the absolute value of the slope of indifference curve actually is), diminishes here as $\sigma_p^2$ is progressively increased for higher $r_p$. While this is the assumption that economists usually make in such studies, this suggests that aversion to risk increases with wealth. The proportion of wealth invested in risky securities should decrease, in that case, as the amount of wealth increases. But this has been a contentious issue\(^\text{20}\).

Consider the results of the Fed’s triennial Consumer Finance Survey, the most recent of which was conducted in 2001\(^\text{21}\), for instance. As shown in Exhibit 5.16, these data show that wealthier families are more exposed to the stocks than the less wealthy families. The proportion of families holding certificates of deposit, where the risk of default is almost nonexistent for up to $100,000 thanks to FDIC (Federal Deposit Insurance Corporation) insurance, appears to move slowest with family wealth, however. Perhaps this suggests that families invest in the stock market only after they have exhausted the other avenues. After all, this survey also revealed that 58% of the total assets of all families in 2001 were held in nonfinancial instruments, mostly in residential and nonresidential properties and equities. As for stock holdings, though, we should note that mutual fund and retirement accounts tend to be dominated by stocks and that, between 1998 and 2001, the relative proportion of those holding financial assets gained slightly, at the expense of the nonfinancial holdings. Curiously, if we group certificates of deposit and bonds as low risk assets and group stocks with the presumably equity dominated mutual funds and retirement accounts as the relatively risky assets, then the ratio median family holdings in these risky to low risk asset groups is actually higher for the poorer 40% of families than for the richest 10%. These data hardly support the possibility that the propensity for bearing financial risks increases with family wealth.

A more compelling reason\(^\text{22}\) why most researchers prefer using the quadratic utility function, however, is that it is completely specified by its
mean and variance regardless of whether the underlying statistical distribution is normal or otherwise. It is positively related to the expected wealth and negatively to the risk in securing that wealth.

Exhibit 5.16: As the results of Federal Reserve Board’s 2001 Consumer Finance Survey show, exposure to stock market rises with family wealth. But the proportion of families investing in stocks versus bonds seems to have remained independent of family wealth. (Source: Federal Reserve Bulletin, January 2003).

<table>
<thead>
<tr>
<th>Classification based on personal income</th>
<th>Poorest quintile</th>
<th>20 — 39.9%</th>
<th>40 — 59.9%</th>
<th>60 — 79.9%</th>
<th>80 — 89.9%</th>
<th>Richest 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Certificates of deposit</td>
<td>10.0%</td>
<td>14.7%</td>
<td>17.4%</td>
<td>16.0%</td>
<td>18.3%</td>
<td>22.0%</td>
</tr>
<tr>
<td>Bonds (also savings bonds)</td>
<td>3.8%</td>
<td>11.0%</td>
<td>15.6%</td>
<td>28.1%</td>
<td>34.2%</td>
<td>42.4%</td>
</tr>
<tr>
<td>Stocks</td>
<td>3.8%</td>
<td>11.2%</td>
<td>16.4%</td>
<td>26.2%</td>
<td>37.0%</td>
<td>60.6%</td>
</tr>
<tr>
<td>Mutual funds</td>
<td>3.6%</td>
<td>9.5%</td>
<td>15.7%</td>
<td>20.6%</td>
<td>29.0%</td>
<td>48.8%</td>
</tr>
<tr>
<td>Retirement accounts</td>
<td>13.2%</td>
<td>33.3%</td>
<td>52.8%</td>
<td>75.7%</td>
<td>83.7%</td>
<td>88.3%</td>
</tr>
<tr>
<td>Insurance</td>
<td>13.8%</td>
<td>24.7%</td>
<td>25.6%</td>
<td>35.7%</td>
<td>38.6%</td>
<td>41.8%</td>
</tr>
<tr>
<td>Other managed accounts</td>
<td>2.2%</td>
<td>3.3%</td>
<td>5.4%</td>
<td>8.5%</td>
<td>10.7%</td>
<td>16.7%</td>
</tr>
</tbody>
</table>

Median family holdings of financial assets:

<table>
<thead>
<tr>
<th>Classification</th>
<th>20 — 39.9%</th>
<th>40 — 59.9%</th>
<th>60 — 79.9%</th>
<th>80 — 89.9%</th>
<th>Richest 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Certificates of deposit</td>
<td>$10,000</td>
<td>$14,000</td>
<td>$13,000</td>
<td>$15,000</td>
<td>$13,000</td>
</tr>
<tr>
<td>Bonds (also savings bonds)</td>
<td>$1,000</td>
<td>$600</td>
<td>$10,500</td>
<td>$41,000</td>
<td>$51,000</td>
</tr>
<tr>
<td>Stocks</td>
<td>$7,500</td>
<td>$10,000</td>
<td>$7,000</td>
<td>$17,000</td>
<td>$20,000</td>
</tr>
<tr>
<td>Mutual funds</td>
<td>$21,000</td>
<td>$24,000</td>
<td>$24,000</td>
<td>$30,000</td>
<td>$28,000</td>
</tr>
<tr>
<td>Retirement accounts</td>
<td>$4,500</td>
<td>$8,000</td>
<td>$13,600</td>
<td>$30,000</td>
<td>$55,000</td>
</tr>
<tr>
<td>Insurance</td>
<td>$3,600</td>
<td>$6,200</td>
<td>$7,000</td>
<td>$12,000</td>
<td>$10,000</td>
</tr>
<tr>
<td>Other managed accounts</td>
<td>$24,200</td>
<td>$36,000</td>
<td>$70,000</td>
<td>$60,000</td>
<td>$70,000</td>
</tr>
</tbody>
</table>

Median family holdings of selected nonfinancial assets:

<table>
<thead>
<tr>
<th>Classification</th>
<th>20 — 39.9%</th>
<th>40 — 59.9%</th>
<th>60 — 79.9%</th>
<th>80 — 89.9%</th>
<th>Richest 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential and nonresidential property, including residential equity</td>
<td>$3,000</td>
<td>$8,400</td>
<td>$12,600</td>
<td>$17,600</td>
<td>$22,700</td>
</tr>
<tr>
<td>Business equity</td>
<td>$122,500</td>
<td>$185,000</td>
<td>$175,000</td>
<td>$249,500</td>
<td>$283,500</td>
</tr>
<tr>
<td>Other nonfinancial assets</td>
<td>$56,300</td>
<td>$35,000</td>
<td>$61,700</td>
<td>$62,500</td>
<td>$100,000</td>
</tr>
<tr>
<td>Other debts</td>
<td>$6,000</td>
<td>$6,000</td>
<td>$10,000</td>
<td>$10,000</td>
<td>$20,000</td>
</tr>
</tbody>
</table>

Median family debt:

<table>
<thead>
<tr>
<th>Classification</th>
<th>20 — 39.9%</th>
<th>40 — 59.9%</th>
<th>60 — 79.9%</th>
<th>80 — 89.9%</th>
<th>Richest 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential and nonresidential property</td>
<td>$28,000</td>
<td>$70,000</td>
<td>$94,900</td>
<td>$107,500</td>
<td>$122,200</td>
</tr>
<tr>
<td>Other debts</td>
<td>$13,300</td>
<td>$23,400</td>
<td>$42,800</td>
<td>$83,500</td>
<td>$126,900</td>
</tr>
</tbody>
</table>

Returning to Exhibit 5.15, we can now see why point C is where on the efficiency frontier our savvy investor would choose to be. It is the only point at which an indifference curve, E(U)2 in this case, is tangential to the efficiency frontier and is, therefore, an optimal point at which utility is maximized for an efficient combination of return and risk.
The applicability and relevance of modern portfolio theory need not be limited to the supposedly esoteric world of high finance. Examples abound of how profitably its rationale can be applied in a miscellany of diverse areas like production diversification and product line diversification. Box 5.1 presents an early example of this type. Japan’s Keiretsu system, or General Electric’s diverse holdings for that matter, too illustrate portfolio diversification.

**Box 5.1: Product Diversification at the Automobile Dealerships**

In an article published in the Journal of Marketing (Spring 1980), T. Marx (“The Economics of Single- and Multiple-Line Retail Automobile Dealerships”) presents an interesting application of the Markowitz efficiency frontier for product-line diversification by automobile dealerships. Single-line dealerships, he argued, minimize the capital investment in sales, staff Training, service facilities, and inventories, but raise the risk of public unacceptability. Despite the economic disadvantage of having to forgo these advantages of single-line dealerships, it is this risk that has motivated multiple-line dealerships. By the early 1980s, for instance, one-half of the thirty thousand and odd automobile dealerships had multiple lines. Using the correlation between various product lines in a region is an effective way to identify products for a multiple-line dealership in that region, this study argues. This idea makes immediate sense to the students of portfolio theory, particularly when we look at the correlation matrix, shown above, for the total U.S. domestic car sales during 1970-87. Extending the argument put forth by Marx, one can also see in this correlation matrix an important factor behind Chrysler’s acquisition of American Motors, Plymouth and Dodge.

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**Investments and Retirement Planning**

Okay, this time we’ll try it your way: 3% in stocks, 2% in bonds and 95% in lottery tickets.”
5.2 The Asset Pricing Model and Theory

5.2.1 Capital Asset Pricing Model (CAPM)

It is thus clear that, in designing a suitable portfolio of risky assets like stocks, determining an optimal combination of returns and risk represents only a partial analysis. Completing it requires looking at the investor’s own risk/reward tradeoffs and matching them with the available market opportunities. This Markowitz model is built on the following assumptions:

- Investors focus on the expected rate of return and on the volatility of a security.
- Investors prefer higher expected returns at lower expected risk, and therefore wish to hold efficient portfolios: those yielding maximum expected returns for the given level of risk, or minimum level of risk for a given level of expected return.
- Investors agree on the probability distribution of rates of return on securities. This ensures a unique efficiency frontier.

The investor in a Markowitz world proceeds systematically in selecting securities that are less than perfectly correlated. The construction of the efficiency frontier is not based on a random selection of securities. Where along this efficiency frontier an investor chooses to be is determined by his or her own utility function. For one investor, for instance, point C in Exhibit 5.15 is the optimum point. Another investor, with different preferences, may prefer another point on the same efficiency frontier. There is no unique combination of risky securities that all investors should prefer, therefore. Instead, each investor may allocate his or her wealth differently among the risky securities, albeit on the same efficiency frontier. That is why Markowitz could not derive a general equilibrium asset-pricing model. The model could only describe the tradeoff between return and risk in the market for securities and in the mind of the individual investor.

The development of the capital asset pricing model (CAPM) began with the work of William Sharpe and John Lintner. Sharpe introduced the concept of risk-free asset in the analysis, whose effects now reverberate throughout the world of investment and capital management. The CAPM adds the following assumptions to those in the Markowitz model:

- There is equilibrium in the security markets (this equilibrium is only partial, the effects of the securities markets on the production sector was ignored, for example, and is a characteristic of the pure exchange economy.)
Investments are divisible, i.e., any size of investment is feasible.

There is a risk-free asset, with a risk-free rate, at which the investors can borrow or lend.

Transaction costs or taxes are ignored.

The \textit{ex ante} expectations about the market as a whole are homogenous and all investors agree on the distribution of rates of return (i.e., this translates into the idea, explored in the next section, that the market is efficient).

Investors are risk-averse and maximize the mean-variance utility functions. They maximize one-period expected-utility-of-wealth and the length of the period (the investment horizon) is identical for all investors.

To derive the Sharpe model, we start by decomposing the portfolio, with expected return $E(r_p)$ and volatility $\sigma_p$ as before, into its market and risk-free components: the market component of risky securities accounts for the proportion $w_m$ of the investor’s wealth while the remaining proportion $w_f (= 1 - w_m)$ of investment is risk-free. Let the expected return and volatility for this risky component be $E(r_m)$ and $\sigma_m$, and those for the risk-free component be $E(r_f)$ and $\sigma_f$, respectively. By definition, then, this risk-free component is characterized by $\sigma_f^2 = 0$. Where would the investor find such an asset? For this we turn to Exhibit 5.17. It summarizes the return and risk statistics for selected financial assets for 1926-1999 period from the CRSP database. Notice how the risk term effectively disappears from the statistics for Treasury bills if inflation is factored in. Here, the standard deviation for Treasury bills, which translates into their risk or volatility, is barely two-thirds of that of inflation for this period whereas the mean return for these bills exceeds the inflation rate. An investment in these assets will retain its value, therefore, carrying little risk of loss.

<table>
<thead>
<tr>
<th></th>
<th>Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nominal</td>
<td>Real</td>
</tr>
<tr>
<td>Large-Company Stocks (S&amp;P-500)</td>
<td>13.0%</td>
<td>9.7%</td>
</tr>
<tr>
<td>Small-Company Stocks</td>
<td>17.3%</td>
<td>13.8%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>6.0%</td>
<td>3.0%</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>5.7%</td>
<td>2.7%</td>
</tr>
<tr>
<td>Treasury-Bills</td>
<td>3.9%</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

\textit{Exhibit 5.17}
Annualized return and risk statistics for selected financial assets for the 1926-2000. Here, return is computed as geometric monthly mean return times 12, the corresponding standard deviation (= monthly standard deviation times $\sqrt{12}$) being the measure of risk.

Plugging the above nomenclature in equations (5.3a) and (5.3b), we then find that, for our portfolio

\[
E(r_p) = r_f + \frac{[E(r_m) - r_f]}{\sigma_m} \sigma_p
\]  

(5.6a)

The graph of \(E(r_p)\) versus \(\sigma_p\) is called the capital market line whose slope, \([E(r_m) - r_f] / \sigma_m\), is the Sharpe-ratio that we discussed earlier, in section 4.2 and Exhibit 2.83. It serves as a measure of the market’s risk-adjusted performance. The capital market line defined by equation (5.6a) is shown as line AMB in Exhibit 5.18. As point M here, with the coordinates \([E(r_m), \sigma_m]\), is the market portfolio, a risk-averse investor would chose a point between A and M on this line, depending on the degree of aversion to risk, whereas a risk-taker might even borrow at the risk free rate to invest in M and therefore choose to be anywhere between M and B or beyond.

Exhibit 5.18:

Investing in a portfolio of risky and risk-free assets means being on the capital market line AMB. The curve XMY here describes the efficient set of risky assets.

This construct has simplified the investor’s choice to one of deciding between the weights in M, the market portfolio, and \(r_f\), so limiting the flexibility allowed by individual preferences. As for the risky assets themselves, the curve XMY describes their efficient set, as this curve is the Markowitz efficiency frontier described in the preceding section. Notice that point M is at the tangency of the efficiency frontier and the capital market
This is the *separation principle* in financial economics. Rather than having to qualitatively evaluate the levels of utility and risk aversion, it decomposes the investment decision into two. The investor first finds the efficient set of risky assets (i.e., curve XMY) using the relevant return, variance and covariance statistics, and then adds risk-free assets to it depending on the desired location on the capital market line.

Let us now look at two points, P and Q, in Exhibit 5.18 and suppose that they represent return and risk on any two portfolios. By construction, both have the same variance or standard deviation but different returns, that for Q being greater than that for P. As P is located below the capital market line here, it has clearly underperformed the market. After all, the market portfolio M not only has a return that exceeds the return for P but also has a standard deviation that is less than the standard deviation for P. Our portfolio Q presents the opposite picture. Notice that it lies above the capital market line. Thus, even though its standard deviation exceeds that of the market returns, its returns exceed the market returns by a proportionately wider margin.

Equation 5.6(a) and Exhibit 5.18 thus provide us with a direct means to evaluate the performance of mutual funds. Exhibit 5.19 illustrates this with a practical example. Here we compare the load-adjusted 5-year average returns on the top mutual fund performers under different categories, e.g., U.S. stock funds (large, mid-cap and small growth, value and blend and the specialty funds like those in communications, financials, health, natural resources, precious metals, real estate, technology and utilities), international stock funds (world stock, diversified emerging markets), bond funds (high-yield, intermediate-term, international, government, multi-sector, short-term) and hybrid funds (domestic and international). Since some mutual funds are ‘load-funds’ while others are ‘no load’ funds, our use of load-adjusted returns creates a level playing field for comparison. Our capital market line has been constructed hereby joining plots for market return (annualized total return on the S&P-500 index) and 90-day Treasury-bill rates. The funds that plot about or above the capital market line here are clearly the market-performers and over-performers while those that plot below this line are the under-performers. The past five years have hardly been the best time for precious metals. It is not surprising to find the best-performing specialty fund in this sector, Vanguard Gold and Precious Metals (VGPMX), as the worst of all funds in our select list of funds in Exhibit 5.19! Notice, also, that the best-performing real estate fund, Delaware Pooled Real Estate Investor Trust (DPRIX), has only performed at or about the market index.
The universe of mutual funds is huge, of course, and has been the subject of innumerable examinations since Jensen’s classic study. Jensen found it possible to distinguish consistently good managers from the persistently bad ones but, overall, these studies suggest that the gains by professional fund managers generally cover the portfolio management expenses. A discussion of these and related matters would be tangential here, however. Our purpose is to show, with the help of Exhibit 5.19, how easy William Sharpe’s seminal work had made it for an individual investor to track the performance of mutual fund or portfolio that he or she may have invested in.

We had, in Exhibit 5.18, defined point M as representing the market portfolio. But the securities markets price individual securities, not the portfolios or indexes comprising them. For instance, the value of the Dow at any given point in time depends on what its component stocks are priced at, not vice versa. We need, therefore, to be able to find how the return and the variance of individual securities in a portfolio are related.

### 5.2.2 CAPM for Measuring Performance:

To compute the expected return $E(r_i)$ on $i^{th}$ security in the market portfolio, we note, in Exhibit 5.18, the tangency of capital market line $AMB$ to the efficient set $XMY$ at $M$ and, therefore, equate the slope $(E(r_m) - r_f)/\sigma_m$ of capital market line with that of the efficient set at this point. This yields the following basic statement of the capital asset pricing model (CAPM):

$$E(r_i) = r_f + \beta_i [E(r_m) - r_f]$$
where $\beta_i = \rho_{im} (\sigma_i/\sigma_m)$ is the measure of the extent to which our $i^{th}$ security’s rate of return moves with that of the market. It is also the measure of systematic or non-diversifiable risk, the component of risk that cannot be eliminated through portfolio diversification. The entire second term on the right hand side of this equation, $[E(r_m) - r_f] \times \beta_i$, is the risk premium on our $i^{th}$ security with systematic risk equal to $\beta_i$.

Equation (5.6a) defines the linear relationship between risk and return by stating that the rate of return from a security comprises (a) the risk-free rate and (b) the adjusted risk premium. The first of these compensates for the time value of money while the second is that security’s $\beta$ (beta) times the market risk premium. For the whole market, $\rho_{im} = 1$ and $\beta_m = \rho_{im} (\sigma_i/\sigma_m) = \rho_{im} (\sigma_m/\sigma_m) = 1$. In practice, $\beta$ is the sensitivity of an equity’s risk premium to the changes in market risk premium, or the slope of the security market line. This is because equation (5.6b) can be written as

$$
\beta_i = \frac{E(r_i) - r_f}{E(r_m) - r_f} = \frac{\text{Risk premium on the security}}{\text{Risk premium on the market}}
$$

By way of illustration, Exhibit 5.20 compares the performances of four stocks for the April 1986 - March 2001 period. With the highest $\beta$ of all the stocks compared here, Microsoft (MSFT) has also given the best returns. The returns on General Motors (GM) and Proctor & Gamble (PG) stocks have been poorer in comparison, but their $\beta$ values are also appreciably smaller.

Exhibit 5.20 is a variant of one of the classic empirical tests supporting the CAPM — that study, by Fama and MacBeth$^{29}$, demonstrated a positive relation between average return and beta. It also illustrates a common method for estimating beta. If, based on equation (5.6c), we regress the observed risk premium $[E(r_i) - r_f]$ for a security or portfolio against the market risk premium $[E(r_m) - r_f]$, then $\beta_i$ is the slope of the resulting linear regression equation. The straight line from such a regression would have an intercept as well, say $\alpha$, at $[E(r_m) - r_f] = 0$. We deliberately set it at zero when computing the data presented in Exhibit 5.20 in order to conform to equation (5.6c). Jensen$^{30}$ introduced $\alpha$ as a performance measure and showed that mutual funds that outperformed the market had statistically significant positive $\alpha$ and those with statistically significant negative $\alpha$ consistently underperformed the market. It is therefore called Jensen’s $\alpha$.

As for the market, the S&P-500 index usually serves as an excellent proxy whereas, as for the risk-free rate, the rate on 3-month Treasury bill is the most commonly used proxy. The alternative is to find a zero-beta security
or portfolio whose expected return, $E(r_i)$, bears no correlation with the market return (i.e., $\rho_{mx} = 0$). Equation (5.6b) would then modify\(^3^1\) to

$$E(r_i) = E(r_z) + [E(r_m) - E(r_z)] \times \beta_i$$ \hspace{1cm} (5.6d)

An updated version of the other classic empirical test\(^3^2\) of CAPM is presented in Exhibit 5.21 where we compare average annual returns on the five asset classes of Exhibit 5.17, covering the 1926-1999 period, with their CAPM-derived values shown as the capital market line. Notice how well the theory matches the observed data!

As security prices fluctuate, so do the corresponding returns and betas. Rather than using the CAPM as a deterministic predictor of market behavior, therefore, we need to use it as a broad stochastic guide to the market. As graphed in Exhibit 5.22, fitting equation (5.6b) to the 1995-2000 data shows that more of the Dow components performed either better than or the same as the broad market, as represented by the S&P-500 index, than underperform it. Despite this scatter of data, however, the pattern seen here is

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
\textbf{Annual Return} & \textbf{Annual Risk} \\
\hline
MSFT & 50.17% & 33.27% \\
HD & 33.74% & 25.72% \\
PG & 23.25% & 21.15% \\
GM & 20.47% & 25.82% \\
\hline
\end{tabular}
\end{center}

Note: Standard deviation is the measure of risk.
what CAPM tells us to expect, i.e., the greater the beta the higher the returns and the risk premiums.

Exhibit 5.21:
Average annual returns for five asset classes of Exhibit 3.18 graphed against their $\beta$ values. Also shown is the capital market line derived from equation (3.6b).

Exhibit 5.22
The Jan 1995 – Dec 1999 monthly data show that most of the Dow components performed better than or on par with the market (open symbols) than underperformed (solid symbols). Star denotes the S&P-500 index and the market line is based on equation (5.6b).

The Dow is an excellent portfolio, of course, and the fact that some of the Dow components may plot below the CAPM’s security market line during some periods does not necessarily render them rejectable. As for the
Dow’s merits as a portfolio, Exhibit 5.23 shows how two of the CAPM-based measures have varied through the history of this index. One of these, the Sharpe ratio, was mentioned earlier in equation (2.13) and graphed for the overall U.S. equities market in Exhibit 2.83. It measures the risk premium that an investor receives in terms of the portfolio risk borne. Treynor ratio is the other measure. Defined as

\[
\text{Treynor ratio} = \frac{[E(r_m) - r_f]}{\beta_I} \tag{5.7}
\]

it measures the risk premium in terms beta, or the portfolio’s systematic risk. These two ratios give similar results if the portfolio is well-diversified such that only the systematic risk remains. A poorly diversified portfolio, on the other hand, would have a smaller Sharpe ratio but a larger Treynor ratio. Note how closely together the two ratios for the Dow have moved in Exhibit 5.23. This justifies our claim that Dow is a well-diversified portfolio.

**Exhibit 5.23:**

*Dow’s Sharpe and Treynor ratios through the twentieth century have moved in tandem, suggesting that it is a well-diversified portfolio. These ratios have been computed in annually rolling bands of 30-year segments here, using equations (2.13) and (5.7).*

5.2.3 CAPM’s Problems and Limitations:

Coming now to the second issue raised above, while it is easy to gauge a security’s or portfolio’s performance relative to the CAPM’s market line, the question of retaining or ejecting underperformers from the portfolio rests on the reliability and consistency of beta as a valid measure of risk. For instance, Exhibit 5.23 shows that investors received better equity premium in the 1950s through 1970s than in the 1980s and early 1990s. Indeed, as the market’s return and risk statistics have varied over time (e.g., Exhibits 2.33
and 2.46), so has the risk premium. But what poses a problem for the CAPM is the fact that there have been protracted periods in history when beta has been a poor measure of the risk premium. Exhibit 5.24 summarizes the results of Fischer Black’s famous study\(^\text{34}\) that showed, for the 1931-91 data, returns below the market line from high beta securities and returns above this line for the low beta securities. Indeed, the data for 1966-91 segment of this study showed statistically comparable returns, about the same as the whole market portfolio, across all beta levels.

Exhibit 5.24

In a study of 1931-91 NYSE stock returns, Fischer Black found a general rise in risk premium with \(\beta\), as the CAPM predicts. The returns across different levels of \(\beta\) did not lie on the market line, however. The numbers here denote a \(\beta\)-based grouping of the stocks. Also, for 1966-91, risk premiums were about the same across all \(\beta\) levels.

The results such as those in Exhibit 5.24 are particularly bothersome because, as we saw in Exhibit 5.21, small company stocks have historically higher betas and returns than the large company stocks. Apparently, and contradicting the CAPM premise, beta alone cannot explain why the expected returns differ. Banz\(^\text{35}\) was one of the first to show that returns are better explained by a firm’s size, measured in terms of the market valuation of its equity, than by CAPM’s beta. Fama and French\(^\text{36}\) have found a firm’s price to book ratio to be an even better indicator of the returns on its stock than the size.

The validity of beta as a measure of risk has thus become a contentious issue for the academics as well as the practitioners. The seeds of the controversy were planted by Roll who argued that, for a theory that acquired its reputation on the claim of easy testability, the CAPM has never been correctly and unambiguously tested and “there is practically no possibility that such a test can be accomplished in the future”\(^\text{37}\).
That the beta of a security itself changes over time was documented by Sharpe himself, one of the principals of what can be now called the SLB (Sharpe-Lintner-Black) one-beta CAPM, in a paper coauthored with Cooper. They estimated betas of securities in the CRSP database for 60-month rates of return for each year from 1931 to 1967, by first ranking and dividing them into ten risk classes and then repeating this procedure for each year. In some cases, almost two-thirds of the securities did not remain in the same risk class, thus pointing to a significant instability of beta. Beta is now known to vary over the business cycle and Jagannathan and Wang have sought to resuscitate the CAPM by advocating that we use more than one beta. They incorporate Mayers' human capital concept in measuring return on aggregate wealth and use time-varying beta and risk premium instead of a single beta over the market’s entire history. Pursuing a similar strategy, Breeden and his associates advocate measuring a security’s risk by its sensitivity to changes in investors’ consumption. Exhibit 5.25 schematically explains this consumption CAPM and shows how it contrasts with the conventional CAPM.

**Exhibit 5.25**

Consumption CAPM defines risk as the uncertainty stocks impose on investor’s consumption of wealth, compared to the standard CAPM in which risk is the uncertainty that stocks bring to investor’s wealth.

5.2.4 The Arbitrage Pricing Theory:

Arbitrage pricing theory (APT) offers an alternative to the CAPM. Unlike the CAPM’s one factor model, that the tradeoff between risk and return is the investors’ only choice, APT envisions a multifactor scenario. An arbitrage situation is one that involves no commitment in capital and yields a positive rate of return. For it to work effectively, capital markets must be perfectly competitive, and the investors must be rational (i.e., prefer more wealth to less wealth).
To derive the basic expression for the APT, let us start by decomposing the expected return on a stock, \( E(r) \), into its supposedly known macroeconomic sources or “factors” and the ubiquitous “noise”, i.e.,

\[
E(r) = a + \sum_{i=1}^{N} \beta_i \times r_{\text{factor } i} + \text{noise} \quad (5.8a)
\]

The factors 1, 2, 3, …, \( n \) could be interest rates, energy prices and the like, and \( \beta_i \) the sensitivity of \( i^{\text{th}} \) security’s return to the corresponding factor. Roll and Ross\(^{44}\) identify five distinct sources of systematic, non-diversifiable risk for a well diversified portfolio — investor confidence, interest rates, business cycle, long-term inflation and short-term inflation — that together explain about 25% of the price fluctuations of an individual company’s common stock. At the level of an individual company, most of the price volatility comes from such company specific risk factors as production, marketing and management risks. A large diversified portfolio of equities, with at least 40 individual companies and not heavily concentrated in any particular one according to these authors, retains only a very small exposure to these company-specific, idiosyncratic, risks but possesses a large amount of exposure to the non-diversifiable common factors (Exhibit 5.26). Also, as the portfolio’s sensitivity to these macroeconomic risk factors is directly proportional to the aggregate of the individual companies in the portfolio, different weightings of the universe of individual companies produce portfolios with varying risk sensitivities.

Based on equation (5.8a), the expected risk premium can be expressed as:

\[
E(r) - r_f = \sum_{i=1}^{n} \beta_i \times (r_{\text{factor } i} - r_f) \quad (5.8b)
\]

Exhibit 5.26

The sources of volatility for individual companies (left) and large and well-diversified portfolios (right).

Source:
Roll & Ross Asset Management
(www.rollross.com)
The idea here is that (a) each source of systematic risk has its own risk and reward and (b) not all these sources or factors carry the same reward/risk ratio at all time. For each $\beta_i \times (r_{\text{factor } i} - r_f)$ in equation (5.8a), therefore, we can construct a suitable portfolio and then monitor, and suitably adjust, the each portfolio’s risk exposure. For instance, if the sensitivity to each of the factors is zero, then it can be seen by plugging in $\beta_i = 0$ in equation (5.8b) that we have essentially a risk-free portfolio. Its expected return would be $E(r) = r_f$, the risk-free rate. Note that any other situation, with $E(r) \neq r_f$, offers arbitrage profit here. If $E(r) > r_f$, you would buy into the portfolio after borrowing at $r_f$ whereas, if $E(r) < r_f$, then you would profit by selling the portfolio to buy the Treasury bills. As against this, constructing a diversified portfolio that is sensitive to the desired factor would give a risk premium proportional to the corresponding sensitivity $\beta_i$.

The case study of nine New York utilities, reported by Elton et al., and discussed by Brearley and Myers is an excellent illustration of the APT in practice. Exhibit 5.27 summarizes their basic data and computations.

Exhibit 5.27: Estimating the Risk Premium by Arbitrage Pricing Theory

<table>
<thead>
<tr>
<th>Factor</th>
<th>Measured by</th>
<th>Factor Risk ($\beta$)</th>
<th>Expected Risk Premium ($r_{\text{factor } i} - r_f$)</th>
<th>Factor Risk × Risk Premium $\beta \times (r_{\text{factor } i} - r_f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield spread</td>
<td>Return on long-term government bonds less that on the Treasury bills</td>
<td>5.10%</td>
<td>1.04</td>
<td>5.30%</td>
</tr>
<tr>
<td>Interest rate</td>
<td>Change in Treasury bill return</td>
<td>~ 0.61%</td>
<td>~ 2.25</td>
<td>1.37%</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>Change in the value of the U.S. dollar against a basket of currencies</td>
<td>~ 0.59%</td>
<td>0.70</td>
<td>~ 0.41%</td>
</tr>
<tr>
<td>Real GNP</td>
<td>Change in the forecast of real GNP</td>
<td>0.49%</td>
<td>0.17</td>
<td>0.08%</td>
</tr>
<tr>
<td>Inflation</td>
<td>Change in forecast of inflation</td>
<td>~ 0.83%</td>
<td>~ 0.18</td>
<td>0.15%</td>
</tr>
<tr>
<td>Market</td>
<td>...</td>
<td>6.36%</td>
<td>0.32</td>
<td>2.04%</td>
</tr>
</tbody>
</table>

The first two columns here show the factors that these authors identified as the ones most likely to affect the prices of utility stocks in the portfolio and their measures. The sixth factor, called market here, was not a direct measure but was included, instead, to account for the portion of the return that could not be explained by the other five factors. Estimates of factor risk and risk premium were made using the empirical evidence for 1978-1990 period and are given in the next two columns whereas the numbers in the last column are merely the products of the preceding two columns. Adding up these numbers in the last column, as suggested by
equation (5.8b), the expected risk premium for this portfolio works out to 8.53%. This means that, with 1-year Treasury bill rate as about 7% in December 1990, the last year covered in this study, the expected return on the portfolio would be 15.53% (= 7% + 8.53%).

The APT certainly captures the market’s reality far more effectively than CAPM’s one-size-fits-all strategy can. Also, by allowing for multiple sources of risk, it enables constructing the portfolios suited for specific needs. But identifying the factors and their relative importance is a task replete with uncertainties. Generally, for instance, macro-economic factors such as surprises in inflation, GNP and investor confidence and shifts in the yield curve explain the changes in security returns reasonably well. But if they really do as good a job of explaining the market’s gyrations as is needed for the APT to work, then we must already have the means to predict the market! The fact of the matter is that we do not even know if these are indeed the “true” factors that we need. Add to this two other problems that we need to contend with. One, adding the number of betas only means compounding the problems we already have with the one-beta case of CAPM. If time-varying betas and risk premiums are what we need to make the CAPM work, then having to seek a multitude of betas for the same time horizon is hardly likely to solve the problem. Two, arbitrage pricing implies the prospects of securing profits without having to commit the capital, and this is accomplished here by constructing zero-beta portfolios. But then, in the stochastic universe of mean-variance optimization that CAPM and APT inhabit, where is the guarantee that the arbitrage pricing of a zero-beta expected return will indeed end up as the arbitrage price of a zero risk premium, once we have introduced such a miscellany of statistical variables?

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5.3 The Efficient Market Hypothesis and its Implications

5.3.1 The Random Walk of Returns

The extensive use of statistics in our narrative so far makes it hard to imagine that it is only in the past five decades or so that statistical analysis has become an indispensable tool in financial economics. Indeed, when Maurice Kendall claimed in 1953 that stock prices follow a random walk, little did he realize that he was pioneering an altogether new era in financial economics. It is not that Kendall was the first ever to have recognized this, however. That distinction should rightfully go to Louis Bachelier except that, despite anticipating by five years Einstein’s seminal work on Brownian motion, Bachelier’s study had largely remained unknown until Kendall’s claim that the daily changes in stock prices are as likely to be positive as negative. As a matter of fact, it was Bachelier, and not Kendall, who had first conceived the concepts of lognormal distributions and geometric mean for the stock prices returns that we have discussed in section 3.3. Nor was the journal of the Royal Statistical Society, London, the first to let statistics muddle its way into economics, by giving Kendall the forum for such thoughts and analyses. That distinction goes to the Journal of the American Statistical Association, in whose pages Halbrook Working had first talked of the random behavior of commodity prices almost two decades earlier. Kendall’s was a pioneering effort, nonetheless, for the avalanche of studies that followed his work eventually led to Eugene Fama’s formulation of the efficient market hypothesis.

What does market efficiency mean and why should it arouse any investor concern or interest? Fama defined an efficient market as one

‘where there are large numbers of rational, profit-maximizers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants’

so that, on average, ‘competition will cause the full effects of new information on intrinsic values to be reflected “instantaneously” in actual prices’. But then, if stock prices indeed adjusted to all the available information as rapidly as this definition demands, then all stocks should be correctly priced at all times. It would then become pointless to seek any overperformers that could be added to a portfolio and identify the underperformers that need to be dumped from it. As we discussed in the context of Exhibit 5.22, not all stocks justify their price all the time.

The question as to what market efficiency is all about boils down, therefore, to what kind of ‘information’ is reflected in stock prices and when.
This has produced the following three forms of efficient market hypothesis of which Fama’s above definition is for the strong form:

<table>
<thead>
<tr>
<th>Weak-form:</th>
<th>The market price of a security reflects the information contained in that security’s price history. For an investor, seeking superior returns entails turning to the fundamental analysis (discussed in the next section), therefore, in order to retrieve the information that will eventually get impounded in the market price.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-strong form:</td>
<td>Market prices not only reflect the past prices but also rapidly adjust to all publicly available information. Seeking superior returns then necessitates anticipating market’s response to the news or announcements on earnings, dividends, stock-splits, mergers, and the like.</td>
</tr>
<tr>
<td>Strong form:</td>
<td>The market price reflects all information that could conceivably be used to determine the ‘true’ value of a stock. As a hyper-efficient market such as this will always price securities fairly, or at their true worth, the quest for superior returns here would rely on the technical analysis that we discuss in the following chapter of past trends and patterns, if at all.</td>
</tr>
</tbody>
</table>

All these definitions start with the same basic observation, that stock prices fluctuate randomly. Even the weak-form denies the presence of any patterns in the stock prices. A look at Exhibit 5.28 will explain what this really means. Two datasets are compared here for the period of one year, or 250 trading days: the S&P-500 index and a sequence of random numbers. For the S&P-500 Index, this Exhibit shows the daily closing numbers from May 3, 2000 to April 30, 2001. The sequence of random numbers has been generated here for a normal distribution with annual mean = 0 and annual standard deviation = $\sqrt{250} = 15.81\%$. These numbers correspond to daily return = 0% and daily volatility = 1%. Notice how hard it would be, without labeling them, to tell one graph from the other.

**Exhibit 5.28:** The pattern of daily price changes for the S&P-500 index for one-year period (May 3, 2000 – April 30, 2001) appears closely mimicked by the random numbers generated with normal distribution (for annual mean = 0% and annual standard deviation = 15.81%).
5.3.2 From ‘Compass Rose’ to Chaos and Fractals

A random walk implies a lack of memory. Thus, if today’s price carries no memory of the past price, then the price tomorrow is unlikely to be determined by the price today. This is precisely what an efficient market is. Here, a firm’s stock is priced at the level commensurate with that firm’s intrinsic worth, in the long run. The challenge is to determine if one can benefit from short-term fluctuations in stock prices, by ‘buying low and selling high’, say. The problem is that the converse is equally likely. As Samuelson argued, asset prices respond to the unanticipated component of news by fluctuating randomly through time. Based on the CAPM, the only trend would then be for either the price to adjust to the level of these fluctuations or for fluctuations to adjust to the price, or both, so that the equity’s risk premium is its beta times the market risk premium. This is an empirically testable postulate. All that we need to do is compare a security’s price on any given day with the corresponding price on the following day or any subsequent day.

Exhibit 5.29 presents such a scatter diagram for IBM’s daily price changes since January 2, 1962, compared to the following day. Had a pattern in the price changes existed here, it would have been reflected in the trend and correlation. Suppose, for instance, that the change in tomorrow’s price followed that of today. We should then find an upward sloping linear trend with

Exhibit 5.29
This graph, comparing one day’s change in IBM’s stock price with that next day from Jan 2, 1962, to Dec 29, 1999, lacks a distinct pattern, and the correlation coefficient of 0.017 carries no statistical significance. The ‘compass rose’ pattern is intriguing, however, even if it only reflects the fact that stock prices change in discrete jumps.

a positive correlation. What if prices fell the next day, to compensate for the rise today, or vice versa? That would show up here as a downward sloping
linear trend and a negative correlation. What we find here, instead, is the total lack of any trend and a correlation coefficient of 0.017 that is not significantly non-zero. The only inference that would be consistent with this picture would be that the price changes here are completely random. This Exhibit shows only the data for IBM. But that is only by way of illustration. One can choose any other security, or an index for that matter, and the results would be no different.

A peculiarity is seen in Exhibit 5.29, in the form of evenly spaced lines radiating from the origin with the thickest lines pointing in the major directions of a compass. The lines at 90° are most readily visible here, followed by those at 45°. Crack and Ledoit\(^\text{57}\), who first detected it, dubbed it the ‘compass rose’ pattern and ascribed it to the discrete nature of stock prices which restricts the returns to a limited number of values. While they found no predictive use of this pattern, it merits our immediate attention because of its following implications for estimation and predictability:

- This pattern is most commonly seen for the ‘high-growth’ or ‘glamour’ stocks that investors often prefer. It appears clearly, according to Crack and Ledoit, if (a) daily stock price changes are small relative to the price level, and occur in discrete jumps of a small number of ticks, while (b) the stock price itself varies over a rather wide range. Subsequent research\(^\text{58}\) shows that these price changes need not really be small.

- This pattern can be perhaps used for improving stock return forecasts, according to Chen\(^\text{59}\), whereas Kwämer and Runde\(^\text{60}\) argued that it adversely affects the statistical testing of deviations from i.i.d. The former result is of interest to most investors, fund managers and annuity providers while the latter should interest option traders and the employees in high growth firms who increasingly receive significant proportions of their wages in stock options. Recall our discussion that normal distribution model is used for analyzing stock returns because these returns are independent and identically distributed.

Contrasted with this caution is the recent argument of Amilon and Byström\(^\text{61}\), that the constraints Chen, Kwämer and Runde have imposed on their models are too unrealistic to yield economically meaningful statistical inferences. The question whether compass rose pattern can be used to identify the “glamour stocks” that we all seek must therefore remain open, as yet.

The question, therefore, is if the empirical data indeed depart significantly enough from the random walk pattern to jeopardize our use of the normal distribution based statistical measures. This brings us to the world of chaos and fractals and their implications to the financial markets\(^\text{62}\). The
problem basically lies in fitting the normal distribution to empirical data on stock returns. Recall our finding, in the preceding chapter, that annual returns fit the normal distribution model better than the monthly returns. But this finding creates a curious problem. Note that the normal or Gaussian model describes a continuous distribution, not a discrete one. Therefore, if a normal distribution is indeed the correct model, then monthly returns should not describe a poorer fit than the annual returns. Instead, the monthly returns are found to be far too strongly peaked but fat tailed than the normal curve.

A probability density function that mimics this situation better than normal distribution is Cauchy density. Its characteristic function \( g(x) \) is

\[
g(x) = \frac{1}{\pi (1+x^2)} \quad \text{for} \quad -\infty < x < +\infty \quad (5.9)
\]

Here, \( x \) is the variable whose statistical distribution we are trying to examine. Obviously, \( g(x) \) is symmetric about \( x = 0 \) where it has its highest value, and tapers off to zero as \( x \to \pm \infty \). As can be seen in Exhibit 5.30 where we compare the normal and Cauchy curves, a major peculiarity of the latter is that it is far more strongly peaked and fat tailed than the normal curve.

It requires no great imagination, therefore, to see in the Cauchy curve the panacea to our problem in seeking to realistically mimic our high peaked but fat tailed monthly geometric returns that the normal curve matches poorly. Benoit Mandelbrot, amongst whose disciples was Eugene Fama, was perhaps the first to appreciate this reality of the statistical distribution of stock price returns.

\[\text{Exhibit 5.30}\]

Compared to the Gaussian or normal curve, Cauchy curve mimics better a high peaked but fat tailed distribution like the monthly geometric returns in Exhibit 2.33. The normal curve here has mean = 0 and standard deviation = 1. Thus, \( x \) here corresponds to \( z \) in Exhibit 2.32. Shaded regions show where the two curves differ.

Then why not use Cauchy curve, instead of the normal curve, to analyze stock returns? The simple answer is convenience. Based purely on the statistical structure, it is quite likely that the distribution of stock market
returns is non-Gaussian. Note that Fama\textsuperscript{65} had himself started with the exploration of non-Gaussian probabilities for price distributions. Suppose the invisible hand that Adam Smith invoked sets the just price for a security and, while we all have a fair idea of what it might be, we do not know what it actually is. In that case, as the meteorologist Lorenz\textsuperscript{66} discovered when he rounded off the input numbers in his iteration equations, even deterministic equations produce chaotic results if the equations are nonlinear. Perhaps this is what that great mathematician Henri Poincaré had in mind when he concluded, at the dawn of the twentieth century, that “small differences in the initial conditions produce very great ones in the final phenomena”. Likewise, it is plausible that, in reality, our nonlinear, geometric, price changes keep looping about an elusive Lorenz attractor and can be better mapped, therefore, by the Cauchy curve. Exhibit 5.31 graphically displays this behavior of a single-point Lorentz attractor.

\textbf{Exhibit 5.31}

Notice how, in this picture of chaos, a dynamic system like a Lorenz system changes over time in 3-D space, with the path or trajectory looping around and around a central attractor, but never intersects itself.

\textit{Source:}

Dr. J. Orlin Grabbe’s homepage at http://www.aci.net/kalliste/chaos1.htm

Clearly, had fitting a curve to the empirical data been our primary goal, then we have erred grievously by using the normal curve. But our goal is to seek the broad patterns that can guide our investment strategies. Using the simple but well-defined lognormal distribution certainly enables this. As is readily apparent in Exhibit 3.30, the resulting discrepancy is marginal when we limit ourselves to 95\% of the probability curve. This is also suggested by a recent study\textsuperscript{67} of the returns on U.S. (S&P-500), U.K. (FTSE-100), German (DAX) and Japanese (Nikkei-225) stock markets, which showed non-linearity in the time of returns but no evidence of chaos. But then, there is no reason why an aggregation of chaotic processes may not turn out to be non-chaotic\textsuperscript{68}.

The random walk premise\textsuperscript{69} of efficient market hypothesis has often led to the notion that selecting the stocks is a skill-less task that is best left to chance. Nothing could be farther from the truth, however. What the ‘randomness’ in statistical distribution\textsuperscript{70} of price changes, whether normal or
lognormal, actually implies here is that the prices randomly drift about an overall trend. History amply testifies to the fact that, as for the broad market, this overall trend itself is an exponential one. How else would you have an inflation-adjusted annual rate of return that has never dipped below 6% over any 30-year period through the history of the U.S. stock market?

The question whether investment professionals can significantly out-perform the random throw of darts is indeed one that an irreverent and inadvertent but nonetheless significant experiment at the Wall Street Journal has explored at length. Box 5.2 summarizes its results. Note that the data presented here do not debunk the dart-throwing strategy altogether. Racking up a 4.5% annual rate of return is somewhat superior to the yield on Treasury Bills, as a matter of fact. But then, this was a period of substantial market, though not exceptional, growth.

5.3.3 Some Empirical Tests of Market Efficiency

Overall, the basic premise of the efficient market hypothesis, that asset prices fluctuate randomly over time, is a reasonably workable idea. True, above-average returns in a given period — a day, a week or a month — sometimes follow similar returns in the preceding period. But then, the predictive power of these patterns is rather weak, and stock returns often display mean reversion over a 3-5 year horizon.
An early evidence that security prices reflect the market’s immediate absorption of relevant news came from the work of Arthur Keown and John Pinkerton. As summarized in Exhibit 5.32, this study found that the stock price of the target company jumps up immediately at the announcement that it is being taken over or bought out but the subsequent days bring no change in this price. There is an upward shift in price in the days immediately preceding the announcement, however, pointing to a gradual leakage of information to the insiders. This is consistent with the efficient market hypothesis because most of the jump in price occurs on the day of the announcement, and no significant change occurs later, i.e., once made public, the information is absorbed fully and immediately. What about the acquiring firms? Their stocks generally fall, by about 10% on average, over a 5-year post-merger period.

Exhibit 5.32
While impending mergers and acquisitions are poorly held secrets, and may give the insiders some excess returns, most of the jump in price occurs on the day of announcement.

Source: Arthur Keown and John Pinkerton.

The merger of America Online and Time Warner is a case in point. At the time the merger was announced (Jan 10, 2000), America Online was the nation’s largest Internet service provider, with over 20 million subscribers and about $163 billion in market capitalization. Time Warner was then the biggest name in the world of traditional media and was valued at about $100 billion. The effect of this announcement on their share prices was marginal, if at all. Exhibit 5.33 graphs these price-paths from Dec 1, 1999 to Jan 31, 2001, a period that covers the initial announcement as well as the FTC (Federal Trade Commission) and FCC (Federal Communications Commission) approvals about a year later. These data show no conspicuous effect of the merger on prices of Time Warner shares, which have fluctuated between $42.5 and $76.05 in this period [we have used the Time Warner Trading Company (TWTC) shares as the proxy for Time Warner shares]. As for America Online, the trend throughout has been one of a continuous
An unexpected decline in share prices. The broad market itself has been rather flat, or in the trading range, in this period and the S&P-500 index has fluctuated between 1265 and 1527.

Exhibit 5.33: The merger announcement of America Online and Time-Warner did little to arrest the falling trend in America Online’s price and may have only given a temporary boost to Time Warner’s price, if at all. The top panel here shows the prices and the bottom panel their daily changes.

Now, the fact that the markets are efficient does not really make it impossible, over time, to log a better performance than the market. Take the example of Warren Buffet, for instance. As shown in Exhibit 5.34, his Berkshire Hathaway fund has consistently beaten the S&P-500 index. Of the three investments compared here, a $7,455 investment on January 1, 1990, would have grown to $28,305 in the S&P-500 index, $37,653 in Fidelity’s Magellan fund, and $68,000 in Buffet’s Berkshire Hathaway fund. Why this odd investment amount of $7,455? This is what the shares of Berkshire Hathaway were priced at on January 1, 1990.

Though not uncommon altogether, cases such as these are rather rare. But, even here, the record is not altogether unchequered. For instance, if you had bought a Berkshire Hathaway share (BRK.a) at its all time high of $83,330 a certificate in June 1998 then, not counting the appreciation in book value, you would have lost 18% on it by April 2001. Comparable investments would have had you up by 13% in the S&P-500 index and 17.4% in Fidelity Magellan.

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Exhibit 5.34:
The managers and funds that have beaten the market consistently for 10 years or longer are hard to find. Here are two of the notable exceptions: Warren Buffett’s Berkshire Hathaway fund and Fidelity’s Magellan fund.

Overall, though, if you expect that professional fund managers with active hands-on management of their funds would get you better returns than what you would receive by passively investing in an index, think again. Mutual funds are often the best examples of active portfolio management. Thus, if active management indeed works, then their average performance should be superior to that of the market index. But innumerable studies of the vast U.S. mutual fund industry have consistently demonstrated their chronic underperformance. Even when performance has been better, and a fund has ‘beaten the market’ so to speak, the excess return over the market has mostly gone into the fund’s management expenses, and seldom into the investor’s pockets.

Strategies (e.g., Dogs of Dow, Fabulous Five) and effects (e.g., the January effect) that often help beat the market too exist, and are discussed in Chapter 4. To the supporters of the efficient market hypothesis, such market “anomalies” only cause satisfaction, not disappointment, however. The fact that they are anomalies reaffirms the notion that, despite periodic excursions, the market always returns to the fundamentals of fairly pricing the equities based on what they are truly worth.

5.3.4 The Efficient Market’s Irrational Exuberance

A problematic issue for the efficient market hypothesis is that of the market bubbles and crashes. Indeed, it is hard to imagine how they could recur if the markets indeed absorbed information efficiently over time so that equities were priced at the levels commensurate with their economic fundamentals. No price-misalignments would then occur, producing the speculative bubble that a subsequent market crash would seek to correct. Bubbles (e.g., Holland’s Tulipmania or Great Britain’s South Sea bubble)
and crashes (e.g., the end of these bubbles, the crashes of 1792, 1929 and 1987 in the U.S.) do occur, however. Take the October 1987 crash, for instance. The first nine months of that year saw a bubble, with 33% appreciation of the S&P-500 index, and this trend got dramatically reversed by the middle of October. Most of the fall occurred in one day, on October 19, when the index fell by 22%, but this was after a 9% decline in the preceding week. The Dow lost 508 points that day, and the overall U.S. stock market lost about $500 billion.

There were signs that could, in retrospect, have forewarned of the disaster almost a week in advance. Three of them particularly stand out — the announcement of one of the largest merchandise trade deficits in the history of the U.S., the possible elimination by the U.S. Congress of the tax benefits of leveraged buyouts, and the likelihood of Fed raising the discount rate.

These problems could not have directly affected markets outside the U.S., however. But, as can be seen in Exhibit 5.35, the stock markets crashed worldwide. The U.S. stock market has the most capitalization of all equity markets worldwide, and accounted for a larger share of the global markets then. Its turmoil is unlikely to have left the other markets unaffected. This global crash was perhaps the domino effect, therefore. But that still begs the question as to why the market, if it is indeed so efficient that equities stay priced about their fundamental worth over time, allowed the prices to rise so high, speculatively, that they would have to come crashing down when the bubble burst. And, if the crash did not drag stock prices down to the levels well below their intrinsic value then we also need to explain how the market recovered so rapidly. Had you invested $10,000 in the S&P-500 index at the market’s pre-crash peak in August 1987, then the crash would have left you almost $3,000 poorer in November 1987 but, exactly two years later, in July 1989, your investment

---

**Exhibit 5.35**

The crash of October 1987 was not limited to the Wall Street, as can be seen from the drops suffered by the stock market indexes world-wide.$^{79}$

<table>
<thead>
<tr>
<th>Local Currency</th>
<th>U.S. Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>– 41.8% – 44.9%</td>
</tr>
<tr>
<td>Austria</td>
<td>– 11.4% – 5.8%</td>
</tr>
<tr>
<td>Belgium</td>
<td>– 23.2% – 18.9%</td>
</tr>
<tr>
<td>Canada</td>
<td>– 22.5% – 22.9%</td>
</tr>
<tr>
<td>Denmark</td>
<td>– 12.5% – 7.3%</td>
</tr>
<tr>
<td>France</td>
<td>– 22.9% – 19.5%</td>
</tr>
<tr>
<td>Germany</td>
<td>– 22.3% – 17.1%</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>– 45.8% – 45.8%</td>
</tr>
<tr>
<td>Ireland</td>
<td>– 29.1% – 25.4%</td>
</tr>
<tr>
<td>Italy</td>
<td>– 16.3% – 12.9%</td>
</tr>
<tr>
<td>Japan</td>
<td>– 12.8% – 7.7%</td>
</tr>
<tr>
<td>Malaysia</td>
<td>– 39.8% – 39.3%</td>
</tr>
<tr>
<td>Mexico</td>
<td>– 35.0% – 37.6%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>– 23.3% – 18.1%</td>
</tr>
<tr>
<td>New Zealand</td>
<td>– 29.3% – 36.0%</td>
</tr>
<tr>
<td>Norway</td>
<td>– 30.5% – 28.8%</td>
</tr>
<tr>
<td>Singapore</td>
<td>– 42.2% – 41.6%</td>
</tr>
<tr>
<td>South Africa</td>
<td>– 23.9% – 29.0%</td>
</tr>
<tr>
<td>Spain</td>
<td>– 27.7% – 23.1%</td>
</tr>
<tr>
<td>Sweden</td>
<td>– 21.8% – 18.6%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>– 26.1% – 20.8%</td>
</tr>
<tr>
<td>U.K.</td>
<td>– 26.4% – 22.1%</td>
</tr>
<tr>
<td>U.S.A.</td>
<td>– 21.6% – 21.6%</td>
</tr>
</tbody>
</table>
would have grown to $11,240, in nominal dollars! Chances are that you
would have blissfully slept through what Wall Street calls the 'Black
Monday' of October 19, 1987, without ever noticing that the market had ex-
perienced a severe crash. Patience does pay, indeed!

Stock prices often stray from their intrinsic worth or the fundamental
value for extended periods of time. As we clearly saw from our examination
of the market’s history, the trend for most of the time has been one of
appreciation. This translates into increasing returns, and makes the stocks
pricier, so that the returns must eventually drop and trigger a concomitant fall
or “correction” in the price. The pricier a stock gets the greater will be its
price-earnings (P/E) ratio, which would drop when the price gets corrected.
This, as can be seen in Exhibit 5.36 where we compare the 10-year averages
of annualized returns and P/E ratios for the S&P-500 index, gives the market
the kind of cyclicity that we first saw in Chapter 3.

Exhibit 5.36: The presence of similar but lagged cyclicities in the graphs of real
returns on the S&P-500 index and the corresponding P/E ratios
suggests that a self-correcting mechanism is built into the market.
Monthly data for the S&P-500 index have been used here. The
returns here are annualized total returns (real) for 10-year holdings
and are shown here in monthly rolling bands

Note the lagged, not coincident, cyclicities in market returns and P/E
ratios in Exhibit 5.36. Overall, high P/E ratios here coincide with the onset of
declining returns and low P/E ratios portend rising returns, as Campbell and
Shiller\(^{80}\) have reported. This is also reminiscent of the negative correlation of
annualized returns for trailing and forward 20-year holdings that we saw in
Chapter 3. The problem is that the cyclicities so pronounced in the two time
series in Exhibit 5.36 match approximately at best. As Exhibit 5.37 shows,
their correlation improves when, rather than the P/E ratio, we use deviations
from the long-term trend in P/E ratios that we discussed earlier. But even this
improvement (coefficient of correlation = 0.58) is not strong enough to make
the P/E a reliable gauge for the likely market returns.
Exhibit 5.37: P/E ratios correlate well with annualized returns (left panel), and this correlation improves when we use the deviations of monthly P/E ratios from the long-term trend (right panel).

It is hard, nonetheless, to label as random fluctuations the cyclicities in the P/E ratio and annualized returns data seen in Exhibit 5.36. In terms of real prices, for instance, the January 1960 – December 1979 period averaged a 1.63% annual drop (standard deviation = 15.39%), compared to an average annual rise of 8.62% (standard deviation = 13.76%) during January 1980 – December 1999.

Is there a fundamental explanation for such rabid changes in prices over protracted periods. We revisit the Gordon growth model, in Box 5.3, to seek an answer. Note that, compared to the steady growth (g) and discount rate (r) model in Exhibit 2.1, where (r – g) is constant, a rising (r – g) scenario translates into a steep drop in the price (P). By the same token, a falling (r – g) environment has the opposite effect of a runaway price spiral. To find what could have produced such swings in (r – g), we only need to look at Exhibit 2.51. Notice how steeply the interest rates generally rose in the 1960s and 1970s, and how they have been falling since the early 1980s. Thus, as Exhibit 5.36 shows, 1960-79 was a period of declining P/E ratios and returns, whereas 1980-99 witnessed rising P/E ratios and returns.

Misalignments such as these between the asset-fundamentals and their market values are hardly limited to the stock markets, however. They particularly afflict the foreign exchange market. In the case of the stock markets, Summers has argued\(^8\) that, for most of the times, the market’s misalignments either gradually build up or unwind. The asset prices deviate appreciably from the fundamentals during these times, therefore, even if the daily price fluctuations reflect market’s immediate response to the relevant new information and mimic the random walk. The reversion to the mean, on this picture, takes 3-5 years, as mentioned before.
1.18 euro/$ and 1-year forward rate (f currency of 11 of the European Union members, has a spot (s

verge towards the present forward rates. Suppose the euro, the common currency of 11 of the European Union members, has a spot (s

expect that future spot rates in an efficient, risk-neutral, market would con-

traders who agree with the implicit rate of the euro’s fall, or expect that the

1970-2000 interest rate cycle clearly affected stock prices dramatically. Note that,

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tuations that make visualization difficult without adding much more to information. The

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\[ g = \text{ROI} \]

and also shows the spread (= Total Return – ROI) of these returns. We have computed \( g = \text{ROI} \) and ROE from Eq. (ii) — with the 10-year Treasury yields, for the January 1970 – December 2001 period, and also shows the spread (= Total Return – ROI) of these returns. We have computed \( g = \text{ROI} \) and ROE from Eq. (ii) — with the 10-year Treasury yields, for the January 1970 – December 2001 period, and also shows the spread (= Total Return – ROI) of these returns. We have computed \( g = \text{ROI} \) and ROE from Eq. (ii) — with the 10-year Treasury yields, for the January 1970 – December 2001 period, and also shows the spread (= Total Return – ROI) of these returns. We have computed \( g = \text{ROI} \)

\[ r = \frac{(D/P) + g}{E/P} \]

or Eq. (i) which we have computed here as \( G \) (\( \text{ROI} \)). This reflects the fact that future cash streams acquire greater weight in a falling interest rate environment (e.g., during 1982-99), as can be gauged from the Gordon growth equation itself, and from Eq. (2.3) in the text, whereas the cash receipts that are to come farther into the future count for increasingly less in present values when interest rates are rising (e.g., during 1970-82). It can be argued, therefore, that price changes can be explained by changes in interest rates and by changing expectations of future growth. Obviously, exuberance is not necessarily irrational. This also shows how efficient the market is in incorporating the secular changes in interest rate regime.

Likewise, as for the foreign exchange rates, we would ordinarily expect that future spot rates in an efficient, risk-neutral, market would converge towards the present forward rates. Suppose the euro, the common currency of 11 of the European Union members, has a spot (\( f_{\text{euro/s}} \)) rate of 1.18 euro/$ and 1-year forward rate (\( f_{\text{euro/s}} \)) of 1.191 euro/$, suggesting euro’s likely fall against the U.S. dollar. It will therefore attract those buyers and traders who agree with the implicit rate of the euro’s fall, or expect that the euro may fall farther. But those traders who expect the euro to rise would

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prefer to wait for a better rate in the spot market, so drying the supply of the currencies. The resulting supply-demand equilibrium for the two currencies for forward trading would justify formulating the rule

\[ f_{\text{euro}\$/\text{s}} = E(s_{\text{euro} / \text{s}}) \]  

(5.10)

where \( E(s_{\text{euro} / \text{s}}) \) denotes spot exchange rate expected at the time corresponding to the forward rate \( f_{\text{euro} / \text{s}} \). This states the expectations theory of exchange rates, that forward rate equals expected future spot exchange rates, and ensures that returns expected on interest-bearing assets in the two currencies are equal. Empirical data paint the opposite picture, however, because spot exchange rates tend to diverge from, not converge to, the initial value of the forward rate.

Overall, the foreign exchange market too shows the fluctuations that broadly mimic a normal distribution. This is clearly brought out in Exhibit 5.38. The distribution of daily changes in U.S. dollar price of ECU, precursor currency to the euro, for Jan 1, 1997 – Dec 31, 1998 period is shown in the top panel here. The bottom panel shows the corresponding distribution of daily changes in U.S. dollar price of euro for the Jan 1, 1999 – March 20, 2001 period.

**Exhibit 5.38**

Price changes in the foreign exchange market too show random fluctuations that, much like the stock returns, reasonably mimic the normal distribution model.

![Frequency vs Daily Change Graph for ECU/US$ and Euro/US$](image)

Much like those in the equities markets, misalignments in the foreign exchange markets too must eventually disappear, as they do, after persisting for protracted periods comparable to the equities markets though. This is not to claim, of course, that we know the economic fundamentals that presumably govern the equilibrium levels that the rates in the foreign exchange market would eventually settle to.

### 5.3.5 Market Efficiency and the Investor’s Choices

Some scholars therefore advocate either discarding the efficient market hypothesis altogether, whether for the equities market or for the foreign exchange market, or modify it so drastically that it can no longer help
understand the market behavior. De Jong et al.\textsuperscript{86} trace this problem to the fact that the agents who trade in the asset markets are of two types: the “noise traders” who operate by reacting to the market hype or noise\textsuperscript{87}, while the “rational traders” operate based on careful analysis of the market fundamentals, price patterns or charts, and the statistical structure. Whether it is due to this noise, or due to inherent limitations of the efficient market hypothesis, there is a rising clamor for behavioral finance\textsuperscript{88} and against the random walk model, the bedrock of efficient market hypothesis. Two questions therefore arise: (a) does it really matter whether the market is efficient or not? and (b) what is an investor to make of these academic debates about market efficiency?

The best answer to the first question comes from Goetzmann\textsuperscript{89}, who identifies the following benefits of an efficient market:

- Price in an efficient market will not stray too far from the true economic price if you allow arbitrageurs to exploit deviations. This will avoid sudden, nasty crashes in the future.
- An efficient market increases liquidity, because people believe that the price incorporates all public information, and are therefore less concerned about paying too high a price.
- Arbitrageurs provide liquidity to investors who need to sell or buy securities for reasons other than betting on changes in expected returns.

The answer to the second question is given in Box 5.4. It reproduces the six lessons of efficient market hypothesis that Brearly and Myers\textsuperscript{90} consider crucial for corporate financial management but are equally valid for individual investors and investment managers.

What if the market is informationally inefficient\textsuperscript{91}? That poses a paradox, as Grossman\textsuperscript{92} has argued. If stock prices reflect all the necessary information then there is no incentive to acquire information. Why would any one either seek or process the information that the market could share, then, and if no one has sought to gather information then how can prices reflect that nonexistent information?

<table>
<thead>
<tr>
<th>Box 5.4</th>
<th>Six Lessons of Market Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Market has no memory. It is therefore futile to try capturing the market’s recurrent cycles of upturns and downturns.</td>
</tr>
<tr>
<td>2</td>
<td>Trust the market prices. Trying to outwit the market can be a risky proposition, therefore.</td>
</tr>
<tr>
<td>3</td>
<td>The market’s assessment of a firm’s securities holds important clues about the firm’s prospects.</td>
</tr>
<tr>
<td>4</td>
<td>There are no financial illusions. Investors are only concerned with the firm’s cash flows and the part of it that they are entitled to.</td>
</tr>
<tr>
<td>5</td>
<td>The do-it-yourself alternative. Whether it is about mergers, or debt-versus-equity financing, investors are unlikely to pay others for what they can replicate themselves.</td>
</tr>
<tr>
<td>6</td>
<td>Seen one stock, seen them all. Investors buy stocks for their risk-reward characteristics, not for any other attributes.</td>
</tr>
</tbody>
</table>
Obviously, there are two dimensions to the informational efficiency that need to be considered in formulating a suitable investment strategy, or sets of them. One is the market’s informational efficiency; and the other is the informational efficiency of investors who may be either well informed, or moderately informed, or poorly informed. A good case can be then made for investment by indexing if the market is efficient. Since prices in such a market reflect all available information, their changes will occur randomly. Passively investing in an index or a mutual fund, such as Vanguard 500 Portfolio that tracks the performance of the S&P-500 index or a fund indexed to the total market, would then make eminent sense. One could have ridden with the NASDAQ-100, for instance, by buying into the ‘cube’ (QQQ) as was mentioned in the previous chapter.

The virtues of passive investing do not stop at the threshold of an efficient market, however. Every year, no more than about one-third of active investors perform either at or above the market’s level, and two-thirds underperform. Therefore, as Steven Thorley\textsuperscript{93} has argued, “if market prices are not efficient and investing is a matter of talent, then the investors in the underperforming majority will tend to be the same from year to year”. If all the investors started indexing as a matter of routine, on the other hand, then a proportionately larger number of them will obviously start performing at the level of the market, so doubling the proportion of market-performers and overperformers. This would certainly be an improvement, overall. As passive investing involves no information-costs, gross and net returns are the same as the market’s, no matter whether the market is informationally efficient or not.

This is shown in Exhibit 5.39 where we conceptually examine the investor’s returns using two variables: market’s informational efficiency and the investor’s use of that information. By using cost of information as zero, we have integrated our information quotient dimension, i.e., whether or not the investor uses the information, with the active versus passive style of investing. Here, a ‘poorly-informed’ investor is merely one who is not extracting and incorporating information into decision-making.

How does active investing fare in this picture? The acquisition and use of information is not a cost-less exercise but the information above and beyond what is already impounded in the price is hard to find if the market is efficient. By definition, such a market also precludes the possibility of above-market returns. This leaves us with gross returns at the market rate or, taking the cost of information out, a sub-market rate for net returns. As Sharpe\textsuperscript{94} has argued, in an efficient market “before costs, the return on the average actively managed dollar will equal the return on the average passively managed dollar” so that “after costs, the return on the average actively managed dollar will be less than the return on the average passively managed dollar”.

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Net returns on investment are defined by the market’s informational efficiency and the investor’s information quotient.

Market is informationally …

<table>
<thead>
<tr>
<th>Well-informed</th>
<th>… efficient</th>
<th>… inefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross return</td>
<td>Above market</td>
<td>Above market</td>
</tr>
<tr>
<td>Cost of info.</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Net return</td>
<td>Below market</td>
<td>??</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moderately informed</th>
<th>Gross return</th>
<th>Cost of information</th>
<th>Net return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>Uncertain</td>
<td>Moderate</td>
<td>Uncertain</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Poorly informed</th>
<th>Gross return</th>
<th>Cost of information</th>
<th>Net return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>None</td>
<td>Market</td>
<td>Market</td>
</tr>
</tbody>
</table>

Investing style is…

Active

Passive

On the other hand, if the market is inefficient, or weakly efficient at best, then looking at the prices and their history alone will do little good. Now there is a real possibility of identifying a Microsoft or AOL in the offering. But this also means saying good buy to no-load mutual funds, selecting stocks from the universe of 7,000 and odd offerings, and timing when to buy and when to sell. There is also room now for designing and using suitable hedging strategies. The potential for rich rewards is tremendous, therefore, but so is the risk of total loss. The gross returns are as likely to exceed the cost of information in the process, as to be dwarfed by it, leaving the net returns open to question.

Recent research, as also the examples presented above, do point to significant gains from active investing, however. Indeed, there is no reason why an informationally efficient market should preclude the success of simplified strategies that can beat the market. Take the example of the ‘Dogs of Dow’ strategy, or its cousin the ‘Fabulous Five’, for instance. As shown in Box 5.5, their 1971-2000 performances have certainly surpassed that of the market by a significant margin. A normal distribution of returns provides room for above average as also sub-average performances, after all. It is not as if we are looking at a statistical distribution with a finite mean but no variance although such a distribution does exist, as an extreme of the Pareto-Levy family of distributions to which a normal distribution model belongs. But we would then have to also contend with the other, scary, extreme with
no mean and an infinite variance. Mercifully, though, such a situation does not really arise here, particularly as the market beating strategies like ‘Dogs of Dow’ do not call for staying with the same stock year after year. Besides, these strategies may not have above market returns if we consider their risk-adjusted returns, transaction costs and tax implications and also note that a company like Microsoft that paid no dividend until recently would have been an outcast from such lists during the period of its most spectacular growth!

Box 5.5: Beating the Market with the Dogs of Dow

Looking for ways to beat the market? Try the “Dogs of Dow”3, or “Fabulous Five”, its junior variant. These strategies combine the growth potential of undervalued stocks with the strength of the 30 of the best known and most successful companies in the world, the components of the Dow Jones Industrial Index. The chart alongside the graph below show their impressive 1971-2000 performance statistics.

How to play these strategies? For the “Dogs of Dow”, select 2nd through 11th of the least priced Dow stocks with the highest yields (2nd to 6th for the “Fabulous Five”), preferably at the bottom of the fall decline, and sell towards the top of the summer rally. For better results, we could try the latter 2-2½ years of the Presidential Term, although the question whether it would work in this new millennium remains to be answered!

Based on these 1896-2001 averaged data, you could certainly improve your yield by selling Dow at the peak of the summer rally in late August-early September and buying back at the fall bottom in late October-early November. The cycle seems to have shifted about a month back in the 1990s, however.

5.4 The Insights from Insights – Concluding Remarks

This chapter has examined diversification across stocks, investment vehicles, sectors of the economy and the national markets. It reaffirmed the indispensability of diversification as a stabilization tool and a mechanism for dealing with the cyclicality and unpredictability of the market.

Basic to the three insights discussed here — portfolio diversification or the Markowitz model, the risk-return correlation models of asset prices, CAPM and APT, and the efficient market hypothesis based on Gaussian distribution of returns — is the assumption that returns are symmetrically distributed. The simplicity and elegance of this assumption certainly makes generalizations possible, but extracts a major cost in that it ignores skewness and kurtosis in the data. While the latter is actually a desired property of the returns, as we have discussed in Chapter 3, the former is not. What would be the point in investing, for instance, if the returns were not biased towards the positive, or displayed skewness? But, if the returns were indeed skewed, then what accuracy can be placed on the assumption of a symmetric distribution of returns?

Since returns show lognormal distribution, Leland’s suggestion that we replace CAPM’s portfolio beta ($\beta_p = \sigma_{pm}/\sigma_m^2$) by a modified risk measure $B_p$, offers an interesting solution to this problem. This risk measure $B_p$ can be written as

$$ B_p = \frac{\text{covariance}\{\mu_p - (1+\mu_m)^b\}}{\text{variance}\{\mu_m - (1+\mu_m)^b\}} \quad (5.11a) $$

compared to the CAPM portfolio beta

$$ \beta_p = \frac{\text{covariance}\{\mu_p - (1+\mu_m)\}}{\text{variance}\{\mu_m - (1+\mu_m)\}} \quad (5.11b) $$

Here, $\mu_p$ and $\mu_m$ denote the mean returns on the portfolio and the market, respectively, coefficient $b = (\mu_m - r_f)/\sigma_m^2$ is the market price of risk, and $\sigma_m^2$ is the variance of market returns. As $B_p \rightarrow \beta_p$ to a first order approximation, when volatilities are small, neglecting the deviations from symmetric model affects long time-horizon more adversely than short time-horizons. The irony is that active management is better equipped to deal with such problems than passive management, although the latter may need it more than the former!

Clearly, this chapter’s detailed look at the market efficiency debate, and the implications that it has on investors’ choices and strategies, affirms that investment in research intended to beat the market has dubious results, but could not conclude that ignorance may prove to be a bliss.
Endnotes for Chapter 5

1 This is because a typical normal distribution is symmetric about the mean, with about one-half of the values on one side of the mean and the remaining on the other side. As was explained through Exhibit 2.26, the fit to this distribution can be gauged from low skewness and kurtosis values, although these higher moments (third and fourth, respectively, mean and standard deviation being the first and the second) themselves are not used for describing the normal distribution model.


4 Specifically, based on equation (3.1b), \( \sigma_p^2 \rightarrow (w_x \sigma_x^2 + w_y \sigma_y^2) \) as \( \rho_{xy} \rightarrow +1 \), \( \sigma_p^2 \rightarrow (w_x^2 \sigma_x^2 + w_y^2 \sigma_y^2) \) as \( \rho_{xy} \rightarrow 0 \) and \( \sigma_p^2 \rightarrow (w_x \sigma_x - w_y \sigma_y)^2 \) as \( \rho_{xy} \rightarrow -1 \).

5 At \( \rho_{xy} = -1 \), \( \sigma_p^2 = 0 \), the condition we need for zero volatility, if \( w_x \sigma_x = w_y \sigma_y = (1 - w_x) \sigma_y \) or \( w_x = \sigma_y / (\sigma_x + \sigma_y) \).

6 This is arbitrary as using Home Depot for stock X and Microsoft for stock Y would make no difference whatever.


12 Some of the other outstanding recent books on the complexities of international finance, investing and related issues are:

- David Eitman, Arthur Stonehill and Michael Moffett: Multinational Business Finance (Addison-Wesley, 1998);
- J. Orlin Grabbe: International Financial Markets (Prentice-Hall, 1996);
- Paul Krugman and Maurice Obstfeld: International Economics – Theory and Practice (Addison-Wesley, 2000);
- Maurice Obstfeld and Kenneth Rogoff: Foundations of International Macroeconomics (MIT Press, 1996);
- Michael Porter: The Competitive Advantage of Nations (Free Press, 1998);


23 Markowitz portfolio optimization is basically a problem in quadratic programming. Basically, the idea here is to compute the proportion of portfolio to be invested in the i-th asset so as to maximize the return $E(r_p)$ given by Equation (3.3a) and minimize the risk or variance $\sigma_p^2$ given by Equation (3.3b). Following are some of the web-sites that provide practical demonstrations of how it is computed:

www-fp.mcs.anl.gov/otc/Guide/CaseStudies/port
www.solver.com/invcenter.htm


26 The derivation of equation (3.6a) is as follows. For the nomenclature used here, equations (3.3a) and (3.3b) can be rewritten as

$E(r_p) = w_m E(r_m) + w_f E(r_f)$

and

$\sigma_p^2 = w_m^2 \sigma_m^2 + w_f^2 \sigma_f^2 + 2 w_m w_f \rho_{mf} \sigma_m \sigma_f = w_m^2 \sigma_m^2$

assuming that $\sigma_f = 0$.

This yields $w_m = \sigma_p / \sigma_m$. As $(w_m + w_f) = 1$, this means that $w_f = 1 - (\sigma_p / \sigma_m)$, so that, writing risk-free rate $E(r_f) = r_f$, we have

$E(r_p) = (\sigma_p / \sigma_m) E(r_m) + [1 - \sigma_p / \sigma_m] r_f$

Equation (3.6a) now follows from rearrangement of the terms in this equation.


The slope of the efficient set is \( \frac{\partial E(r_p)}{\partial \sigma_p} = \left[ \frac{\partial E(r_p)}{\partial w_i} \right]/\left[ \frac{\partial \sigma_p}{\partial w_i} \right]. \)

Now, if \( w_i \) is the proportion of the market portfolio invested in security \( i \), then

(a) \( E(r_p) = w_i E(r_i) + (1 - w_i) E(r_m) \) and

(b) \( \sigma_p^2 = w_i^2 \sigma_i^2 + (1 - w_i)^2 \sigma_m^2 + 2 w_i(1 - w_i) \rho_{im} \sigma_i \sigma_m. \)

We now differentiate \( E(r_p) \) and \( \sigma_p^2 \) with respect to \( w_i \) and note that, in reality, \( w_i = 0 \) because our \( i \) security could not exist outside the portfolio \( M \) if it is indeed the market portfolio. This gives

\[
\frac{\partial E(r_p)}{\partial \sigma_p} = \left[ \sigma_m/(\sigma_{im} - \sigma_m^2) \right] \times \left[ E(r_i) - E(r_m) \right]
\]

which must be equated with the slope \( \{E(r_m) - r_f\}/\sigma_m \) of equation (3.6a) at \( M \), i.e.,

\[
\left[ \sigma_m/(\sigma_{im} - \sigma_m^2) \right] \times \left[ E(r_i) - E(r_m) \right] = \{E(r_m) - r_f\}/\sigma_m.
\]

On cross-multiplying and rearranging the terms, this gives equation (3.6b).


Lognormal distribution is the normal distribution of logarithmically transformed data, as in Exhibits 2.33 and 2.37 where we have fitted normal distribution curves to the geometric monthly and annual data, respectively. Interested readers may find the book by J. Aitchison and J.A.C. Brown (*The Lognormal Distribution*, Cambridge University Press, 1957) to give perhaps the most exhaustive exposition of this distribution.


Fundamental analysis, discussed in the following section, seeks to identify undervalued and overvalued securities by analyzing such fundamental information as earnings, asset values, business prospects, and the like.

As discussed in the chapter that follows, technical analysis charts the historic price-path to identify the patterns that would indicate its future course, e.g., MACD (moving average convergence-divergence), momentum etc.

With standard deviation $\sigma_{\text{daily}}$ for the daily data as 0.01, the annualized (=250 days) standard deviation $\sigma_{\text{annual}} = \sigma_{\text{daily}} \times \sqrt{250} = 0.01\times\sqrt{250} = 0.1581$ or 15.81%.


J. Orlin Grabbe’s 7-part essay, “Chaos and Fractals in Financial Markets”, available at his home page located at the URL: http://www.aci.net/kalliste/homepage.html, provides a fascinating introduction to this otherwise complex topic. See also
Jess Benhabib: *Cycles and Chaos in Economic Equilibrium* (Princeton Univ. Press, 1992);
Paul A. Glendinning: *Stability, Instability and Chaos* (Cambridge Univ. Press, 1994);


The fact that variances and betas have varied over time makes using the random walk model, which demands stationarity, a gross approximation. A better alternative is to use the less restrictive martingale model. In this case, $r_{t+1} = r_t + \varepsilon_t$, where $\varepsilon_t$ is the martingale difference. See, for instance, Stephen LeRoy: “Efficient Capital Markets and Martingales”, *Journal of Economic Literature*, vo. 27, pp. 1583-1621 (1989).


This has been one of the most researched areas in empirical finance and all the studies have generally come out with the same finding, a chronic underperformance of mutual funds as a group. One of the earliest studies was by M.C. Jensen: “The Performance of Mutual Funds in the Period 1945-64”, *Journal of Finance*, vol. 23, pp. 389-416 (1968). Subsequent studies include
Ray Ball ("Anomalies in Relationships between Securities’ Yields and Yield-Surrogates", Journal of Financial Economics, vol. 6, pp. 103-126, 1978) was the first to use this phrase to describe departures from the efficient market behavior.


This is the so called CNBC or CNN effect: notice how the stocks mentioned favorably on CNBC or CNN spike temporarily, but then revert to their usual level soon after.


William Goetzmann: An Introduction to Investment Theory (http://viking.som.yale.edu/will/finman540/classnotes/class8.html)


Steven Thorley: “The Inefficient Market Argument for Passive Investing” (http://marriotschool.byu.edu/emp/srt/passive.html)


Adapted from Klaus Schredelseker: “On the Value of Information in Financial Markets – A Simulation Approach”


Leland shows that this risk measure $B_p$ is given by

$$B_p = \frac{\text{covariance}[\mu_p - (1 + \mu_m)^2]}{\text{variance}[\mu_m - (1 + \mu_m)^2]} = \frac{[\exp (\mu_p - \mu_m + 0.5\sigma_p^2 - 0.5\sigma_m^2)]}{[\exp (\mu_m - (1 + \mu_m)^2)]} \frac{\exp(-b\sigma_{pm}) - 1}{\exp(-b\sigma_m^2) - 1}$$

Here, $\mu_p$ and $\mu_m$ denote the mean returns on the portfolio and the market, respectively, while $\sigma_p$ and $\sigma_m$ denote the corresponding variances and $\sigma_{pm}$ the covariance of portfolio and market index returns, and the coefficient $b$ is the market price of risk that amounts, in continuous time, to $b = (\mu_m - r_f)/\sigma_m^2$. As the CAPM beta, $\beta_p$, is given by

$$\beta_p = \frac{\exp (\mu_p - \mu_m + 0.5\sigma_p^2 - 0.5\sigma_m^2)}{\exp (\sigma_p^2) - 1} \frac{\exp(\sigma_{pm}) - 1}{\exp(\sigma_m^2) - 1}$$

the first-order Taylor series expansion [$\exp(x) = 1 + x$] yields

$$\frac{B_p}{\beta_p} = \frac{[\exp(-b\sigma_{pm}) - 1]/[\exp(\sigma_{pm}) - 1]}{[\exp(-b\sigma_m^2) - 1]/[\exp(\sigma_m^2) - 1]} = \frac{(-b\sigma_{pm})}{(-b\sigma_m^2)} \frac{\sigma_m^2}{\sigma_{pm}} \approx 1$$

Thus, for small volatilities or short time-horizons, $B_p \rightarrow \beta_p$, so that CAPM’s mismatches have no adverse effects on the portfolio, as in the case of portfolio managers, but the differences between theory and reality can be substantial when the time-horizon is long, as in the case of the long-term investors!